fundamentals of prestressed concrete design

PRECAST/PRESTRESSED CONCRETE INSTITUTE

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Basic revision of this publication became necessary with the adoption of Building Code Requirements for Reinforced Concrete (ACI 318-63) in 1963 and this revision was primarily accomplished by Tom D'Arcy, a structural engineer, while serving as Publications Director on the PCI staff.

This revision has been reviewed by A. H. Gustaferro, Chairman of the PCI Technical Activities Committee, by Messrs. Janney and Elstner, by several representatives of the PCA Structural Bureau and by H. Kent Preston of CF&I-Roebling. PCI appreciates the contributions and additions made by the above individuals, Paul Mast and John Sbarounis of PCA, and by Mario Suarez of Stressteel, Inc.

Final editing and assembly was accomplished by the staff of the Prestressed Concrete Institute.

Examples are based on out-dated codes and specifications. When applied to design problems, use basic principles with codes and specifications currently in effect.

INTRODUCTION

The purpose of this publication is to acquaint practicing engineers with the fundamental principles of designing prestressed concrete structural elements. In order to provide the reader with an adequate background so that he may feel qualified to make use of prestressing as a possible solution to some of his daily problems, it must be assumed that his grasp of structural engineering fundamentals is sound.

Five broad areas important to prestressed concrete design are covered. First, the properties of the two basic materials -- concrete and steel, are discussed, emphasizing the physical properties that affect prestressed concrete. Second, the two primary design considerations, flexure and shear, are explored. Third, typical design examples with respect to buildings and bridges are discussed. Fourth, the salient features of the prominent guiding documents and codes are illustrated and discussed throughout the text. Fifth, general considerations and factors affecting the design of prestressed concrete such as connections and tolerances, are presented.

Prestressing has been defined as the intentional creation of permanent stresses in a structure or assembly, for the purpose of improving its behavior and strength under various conditions of load.

Prestressed concrete has been described as a structural concept that combines the best of two well known construction materials -- concrete and steel. Concrete, one of our most economical structural materials, is capable of resisting relatively high compressive stresses. However, its tensile strength is only 10 to 15 per cent of its compressive strength. In steel we have a material that is strong in tension. Prestressed concrete combines these materials in their most effective capacities.

One way to increase efficiency is to use superior basic materials. Typical prestressed concrete members do this by using concrete that is approximately twice as strong as is usually used and steel that is approximately six times stronger than normal reinforcing. These superior materials also have properties which are necessary to prevent the loss of the prestress force. These properties are discussed later.

But just using superior materials is not enough -- prestressing combines them in the most efficient manner. By stretching the steel before it is bonded to the concrete (prestressing) compressive forces

are placed in the concrete. Thus the steel which is strong in tension is stretched, and the concrete which is strongest in compression is precompressed. And if the steel and the resultant compressive portion are located in the member where tensile forces occur under load, we have used these materials in the most efficient manner and have produced a prestressed member.

MATERIALS

CONCRETE

Concrete is a heterogeneous mixture of sand, gravel, cement and water, plus air, salts, fine inert materials and other additives or admixtures which modify the characteristics of concrete. The properties of the aggregate may vary over a wide range but still produce desirable concrete for structural purposes. The cement can be manufactured from a variety of clays combined with numerous types of calcareous materials. It is inevitable, therefore, that the physical properties of concrete will vary over wide ranges as compared to other structural materials. For instance, the modulus of elasticity of steel is approximately 29×10^6 psi whether it is hot rolled, low carbon steel or whether it is a high strength, cold rolled, alloy steel. On the other hand, the modulus of elasticity of concrete may vary from 1.5 to 7.0×10^6 psi. Average properties should be compared with local concretes, and equations or

numbers should be modified accordingly.

Economy is a primary advantage of concrete because gravel and sand or other aggregates as well as cement are available abundantly in nearly all localities. It is essential that the designer be familiar with the local materials. A designer may insist upon 9000 psi concrete with a modulus of 7×10^6, but if it requires a 300% increase in concrete costs, the judgment of such a demand may be subject to question.

The basic physical properties of concrete and prestressing steel must be understood before the load-carrying behavior of prestressed concrete attains meaning, and knowledge of the load-carrying behavior is an important prerequisite to intelligent design of prestressed concrete.

The properties of concrete with which one should be familiar before attempting to make use of prestressed concrete design are:

a. compressive strength

b. character of the stress-strain relationship

c. modulus of elasticity

d. creep and shrinkage

e. tensile strength

Compressive Strength

Compressive strength must be established in design so that the limiting values of working stresses may be set and to a lesser extent

so that the load-carrying capacity of prestressed units might be predicted.

The manufacture of concrete, particularly in the production of prestressed concrete, must be properly controlled to insure not only quality, but also uniformity. Since the mechanical properties of concrete are related to the compressive strength, this property is the prime consideration in the control of the finished product.

Too often designers and producers feel that if strength requirements are met or exceeded the job is successful. Controlling strength is equally as important as meeting strength. In conventionally reinforced concrete beams, excessive concrete strength could be considered beneficial. In prestressed concrete wide variations in strength, even though in excess of the design minimum, may produce considerable variations in camber and deflection which must be considered undesirable. It becomes obvious that prestressed concrete demands close supervision for accurate control. Proper inspection of materials and plant operation either by the producer or buyer is an important means of achieving the uniformity demanded of prestressed concrete. Although plant inspection is a subject in itself, the need for it is emphasized here. [1*]

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* Numerals correspond to references in the bibliography.

Concrete strengths for prestressing should not be less than 4500 psi except for unusual circumstances. From the design viewpoint higher strengths are more desirable, usually reducing the required volume of concrete. Since concrete costs increase as strength increases, the choice of design strength generally depends upon economics. In most localities 5000 psi has become the accepted normal design strength. However, there is reason to believe that 7000 psi may be an economical strength in plants where such strength can be met and controlled. In time the improvement of equipment and production techniques will bring about an increase in the normal design strength. A few years ago the normal design strength was 3500 psi; 20 years ago 2500 was considered good concrete. In the forseeable future designs requiring concrete strengths of 10,000 psi will not be considered unreasonable.

Concrete strength depends upon many factors such as mix design, placement, curing, materials and control. Space does not permit covering these subjects in this document. However, the strength required in prestressed concrete demands proper mix design and proper control of mixing, placing and curing.

The use of lightweight aggregates in structural concretes, especially concrete for precast structural units, is increasing. There is a tendency to associate lightweight aggregates with low strength

concretes. Many modern lightweight aggregates are used to produce the high strength required for prestressed concrete economically. Knowledge and control, are important to the intelligent use of lightweight structural concretes.[2]

Admixtures are sometimes added to concrete mixes to modify the physical properties of the concrete. When used properly, admixtures can be advantageous. When used without proper control, they can be a source of trouble. Many prestressed concrete producers find the use of water reducing retarders or plasticizing agents of high quality beneficial to placement and early handling. In addition proper air-entrainment is beneficial to prestressed concrete which is to be exposed to freezing and thawing. Calcium chloride or admixtures containing calcium chloride should not be used.

Stress-Strain Relationship

An accurate configuration of the stress-strain curve for concrete has been developed in recent years and has been substantiated by laboratory tests. Although future research may modify or refine the recent work, what is now known provides an ample basis for design.

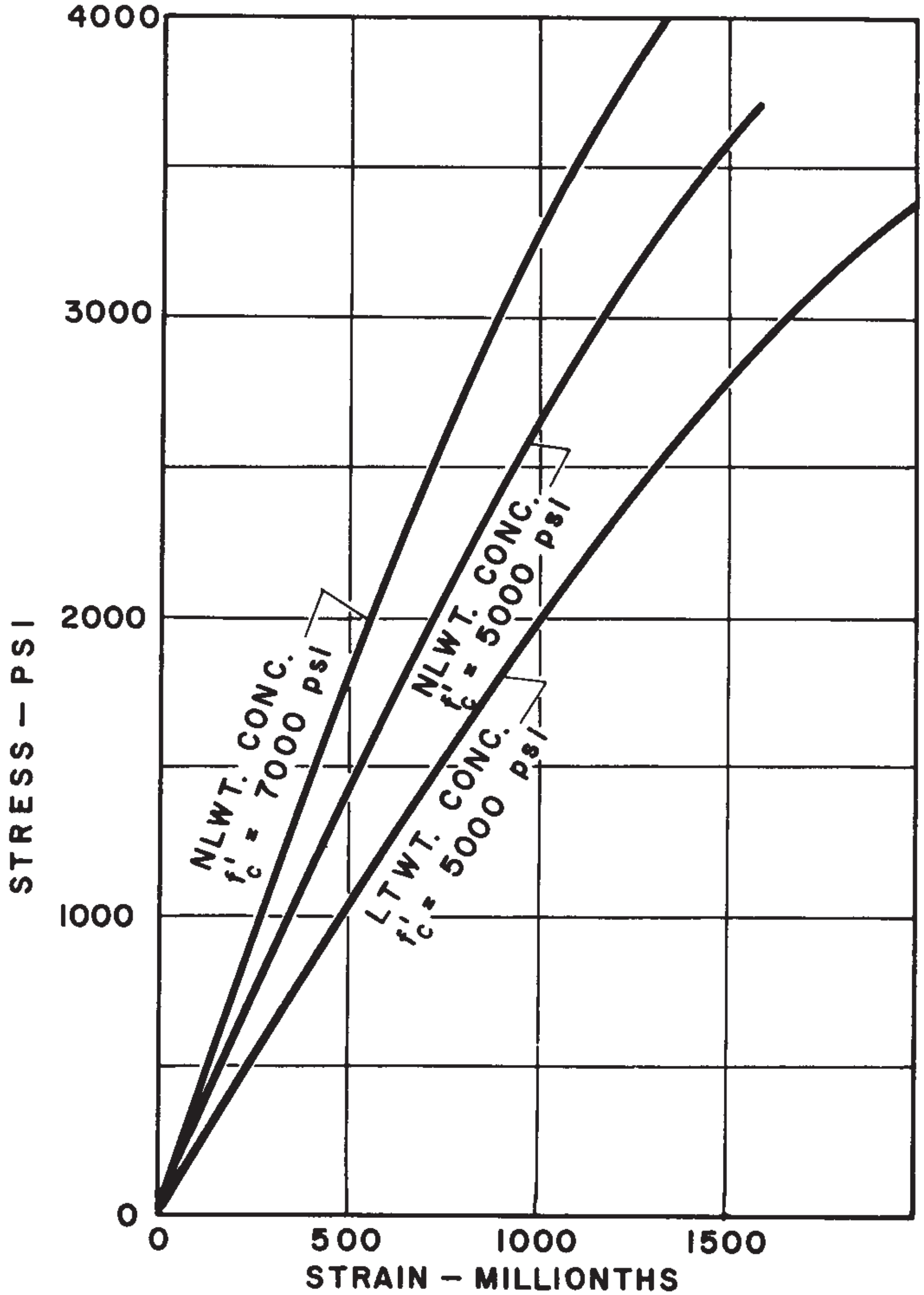
A stress-strain curve obtained from an ordinary cylinder test is

shown in Fig. 1. Failure occurs at a unit strain of about 0.002. Measurements of strains on test columns and test beams indicate much higher compressive strains in the concrete which fails in compression.

The stress-strain relationship as determined by Hanson, Hognestad and McHenry[3] is presented in Fig. 2. This work has revealed some interesting facts about the stress-strain relationship. A generalized stress-strain curve is shown in Fig. 3 indicating the various physical constants which are important in design such as modulus of elasticity, E_c*; the maximum stress as related to a test cylinder, $k_3 f'_c$; the strain at maximum stress, ϵ_o; ultimate strain, ϵ_u; the area under the curve, $k_1 k_3 f'_c \epsilon_u$; and the location of the centroid $k_2 \epsilon_u$. The test work indicates that all of these constants are a function of concrete strength alone - except ϵ_o which remains fairly constant. (Although strength, mix design and age were variables in the investigation, type of aggregate was not a variable.) The relationship of these constants to concrete strength are shown in Fig. 4. More recent tests indicate that these relationships are different for lightweight aggregates. However, it will be shown in the discussion on flexure that the differences are of minor significance.

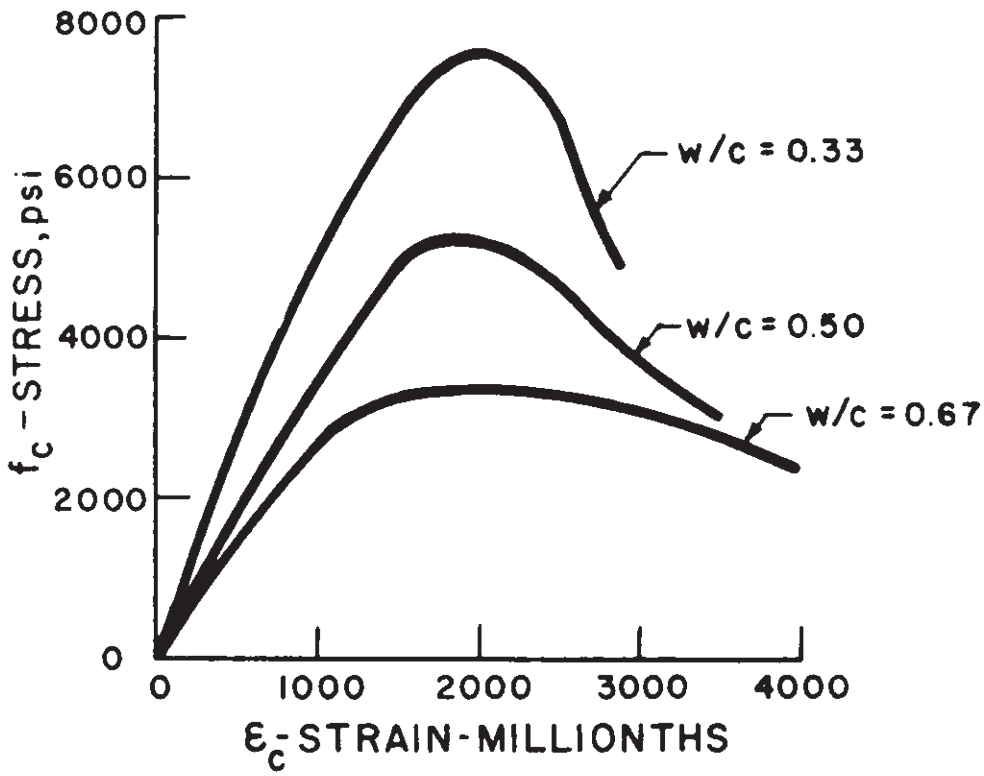
Although the work by Hanson, Hognestad and McHenry has led to the

* For notations see listing at end of text.



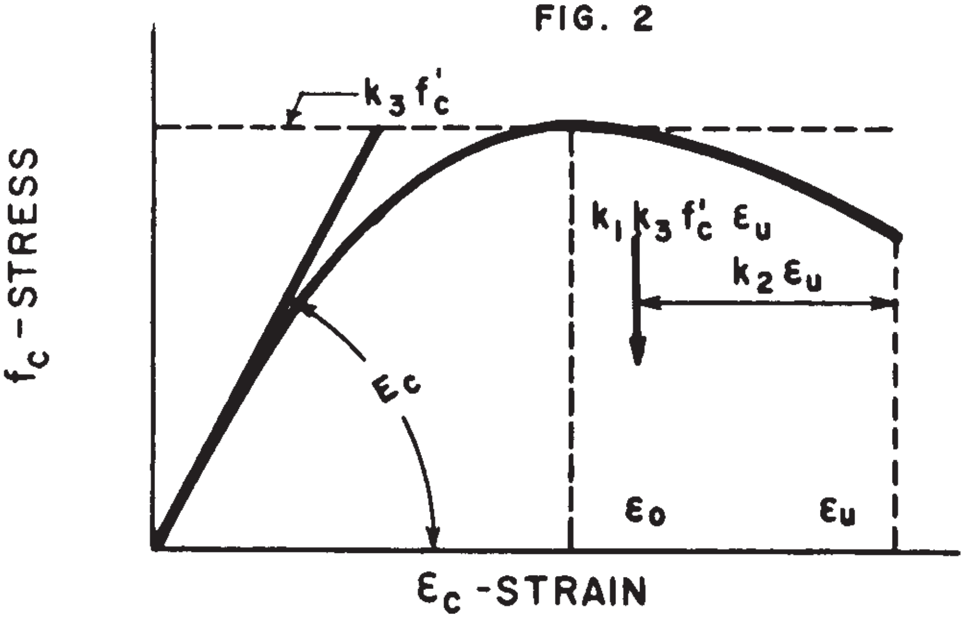
TYPICAL STRESS – STRAIN CURVES FOR CONCRETE

FIG. 1



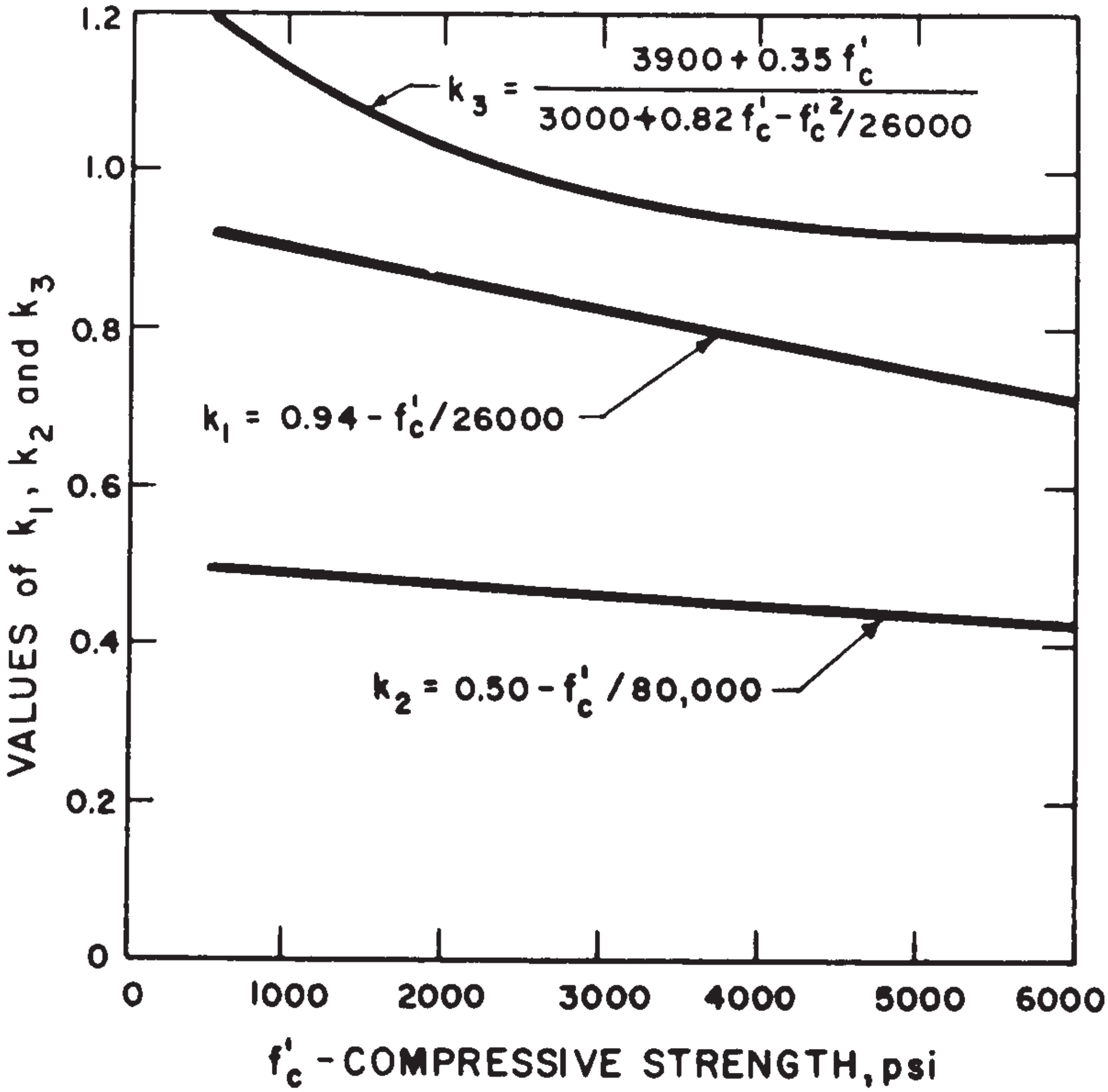
EXTENDED STRESS-STRAIN CURVES
FOR CONCRETE

FIG. 2



GENERALIZED STRESS-STRAIN CURVE
FOR CONCRETE

FIG. 3



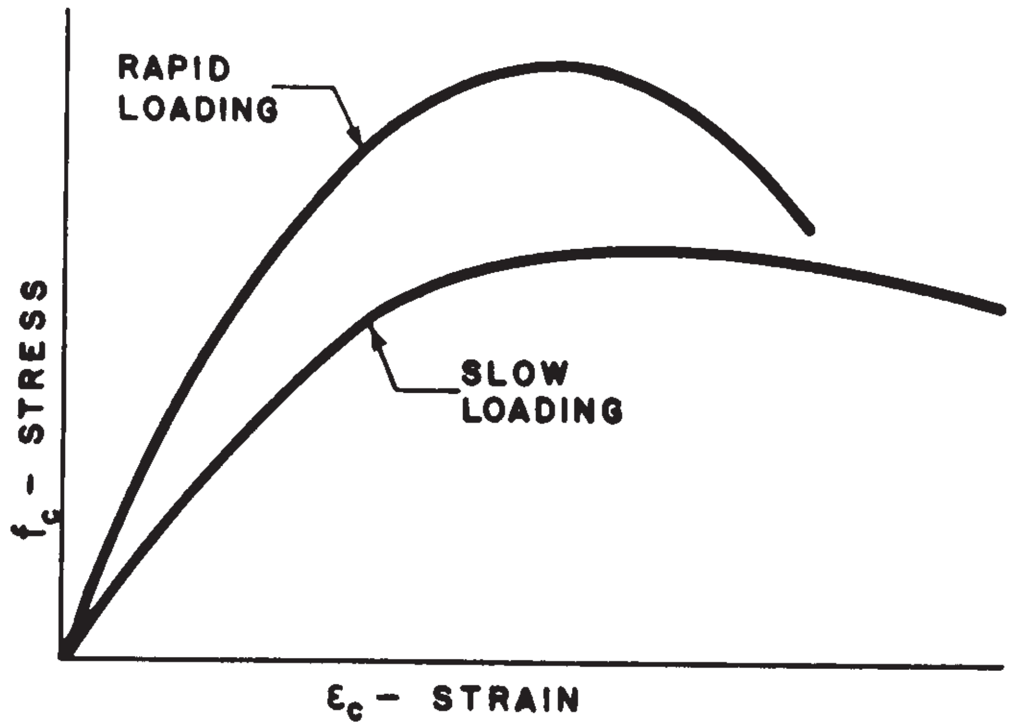
ULTIMATE STRENGTH FACTORS FOR CONCRETE

FIG. 4

first clear understanding of the true stress-strain relationship of concrete, the full picture has not been completed. Of primary concern is the stress-strain curve under long time loading conditions which was not a variable in these tests. Present knowledge of creep permits the assumption that the long time stress-strain curve will be different; the initial slope of the curve will be decreased, the maximum strength will be lowered, and the maximum strain will be increased, Fig. 5. However, experience and laboratory tests indicate that creep has little effect upon strength of unrestrained structural members. Therefore, until research further advances our knowledge, designs must be based upon the short time stress-strain relationships while problems associated with creep must be solved on the basis of sound engineering judgment.

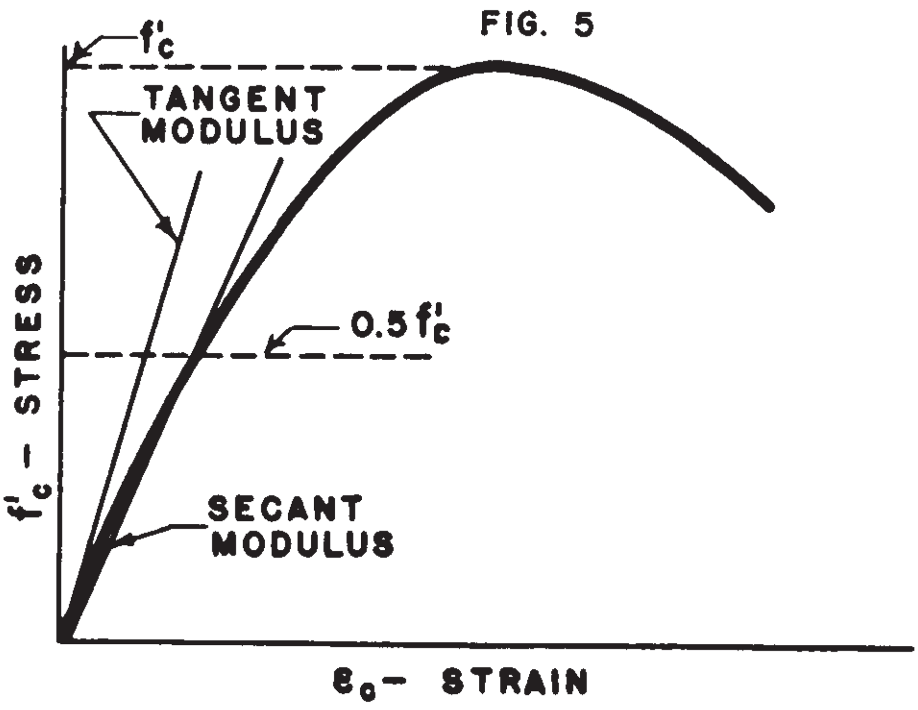
Modulus of Elasticity

Since concrete is not an elastic material, that is, the stress-strain curve departs from a straight line relationship at low stresses, a modulus of elasticity which is a straight line relationship cannot be established. Fig. 6 indicates two moduli, the tangent modulus and the secant modulus (generally determined at a stress of $0.50f'_c$ as shown). For higher strength concretes the stress-strain curve tends to approach a straight line relationship for a larger range of stresses so that the two moduli become nearly identical. This condition is



EFFECT OF LOADING RATE
ON CONCRETE STRESS-STRAIN CURVE

FIG. 5



DEFINITIONS OF ELASTIC MODULUS

FIG. 6

generally the case for concrete encountered in prestressed concretes.

A secant modulus determined at $0.50f'_c$ is normally used for design purposes. Various equations have been proposed to estimate the modulus:

1. $E_c = W^{1.5} 33 \sqrt{f'_c}$ (ACI 318-63 - Building Code Requirements) (1)

2. $E_c = \dfrac{6 \times 10^6 f'_c}{f'_c + 2000}$ (Jensen's Equation) (2)

3. $E_c = 1.8 \times 10^6 + 460\, f'_c$ (Lyse's Equation) (3)

4. $E_c = 1.8 \times 10^6 + 500\, f'_c$ (ACI 323 Rec. Practice) (4)

5. $E_c = 60,000 \sqrt{f'_c}$ (University of Illinois) (5)

6. $E_c = 33 \sqrt{W^3 f'_c}$ (Adrian Pauw) (6)

Where W = unit weight of concrete, $lbs/ft.^3$

ACI 318-63 adopted Adrian Pauw's equation (No. 6) to provide a single equation suitable both for normal and lightweight concretes throughout the presently utilized range of compressive strengths.

The second, third and fourth equations are reasonably accurate for average concretes of average materials. Predicted moduli tend to be too high when f'_c exceeds 5000 psi.

The fifth expression reasonably predicts moduli over the full range of concrete strength but is primarily intended for normal weight concretes. Equation (1) reduces to Equation(5) when W = 150 $lbs/ft.^3$.

Creep and Shrinkage

Creep and shrinkage present an advantage and a disadvantage to prestressed concrete. They reduce the prestressing force. On the other hand, creep prevents concrete from being a brittle material which would shatter when subjected to a high concentration of stress at any point. Creep allows the high stress at one point to flow to nearby areas, thus relieving the concentrations. The advantage of this stress relieving outweighs the disadvantage of prestress loss, provided the creep does not produce undesirable camber, deflection or stresses in fully or partially restrained members.

Creep is defined as the increase in strain with time under constant load. At usual working stresses creep is directly proportional to the applied unit stress. The proportionality no longer exists under overload conditions, but the stress-strain relationship at high stress has not been established.

Drying shrinkage strains are defined as strains resulting from volume changes which take place with time due to the loss of water in the concrete. Shrinkage occurs without stress unless a member is restrained from movement.

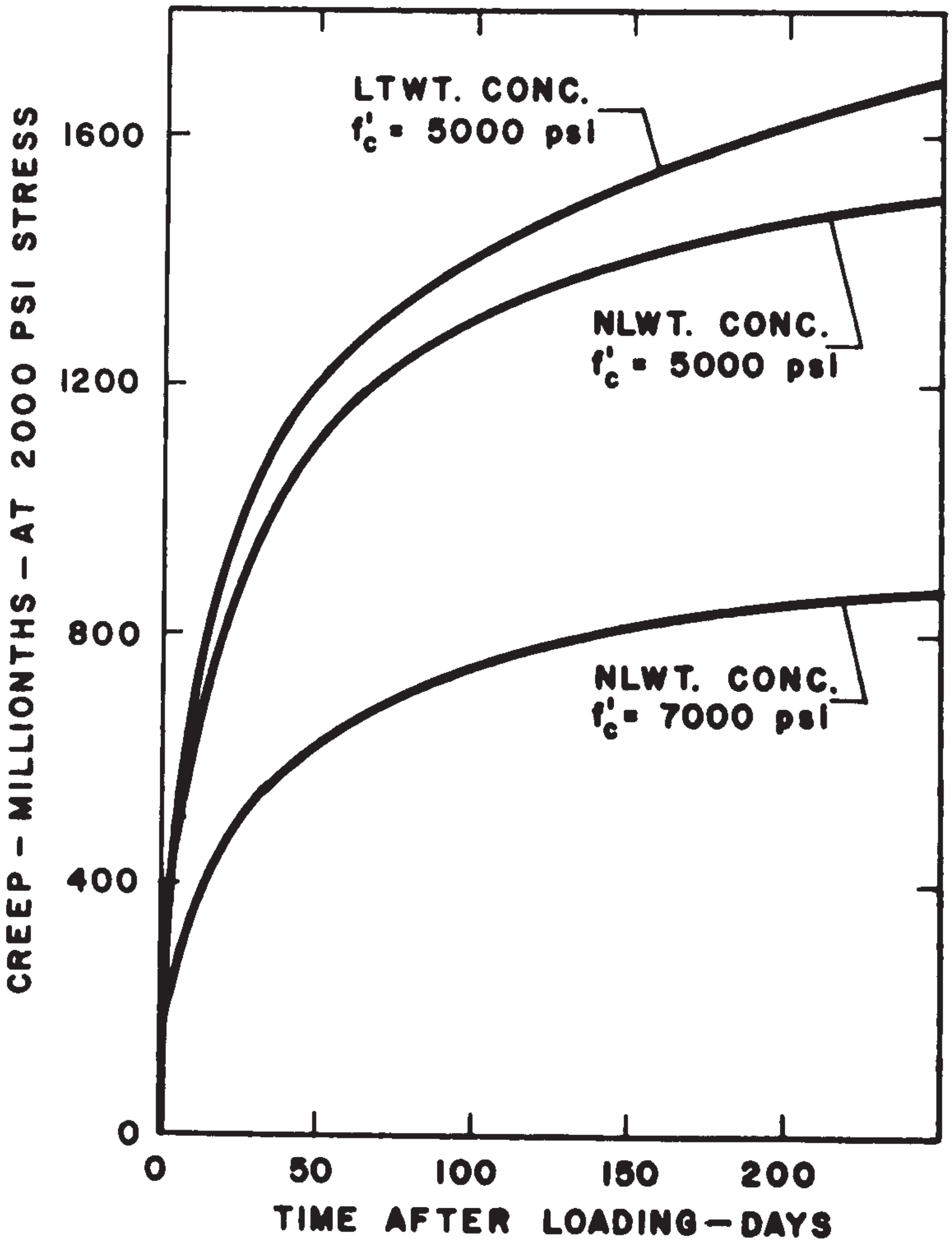
A strain-time curve for both creep and shrinkage is exponential in

character. Large strains occur at early ages, decreasing in rate as time increases. Strains continue to increase for an infinite length of time, although for design purposes the values predicted for from 6 months to 1 year are sufficient.

Factors tending to increase creep and shrinkage are high water-cement ratio, high slumps, soft aggregates and improper curing.

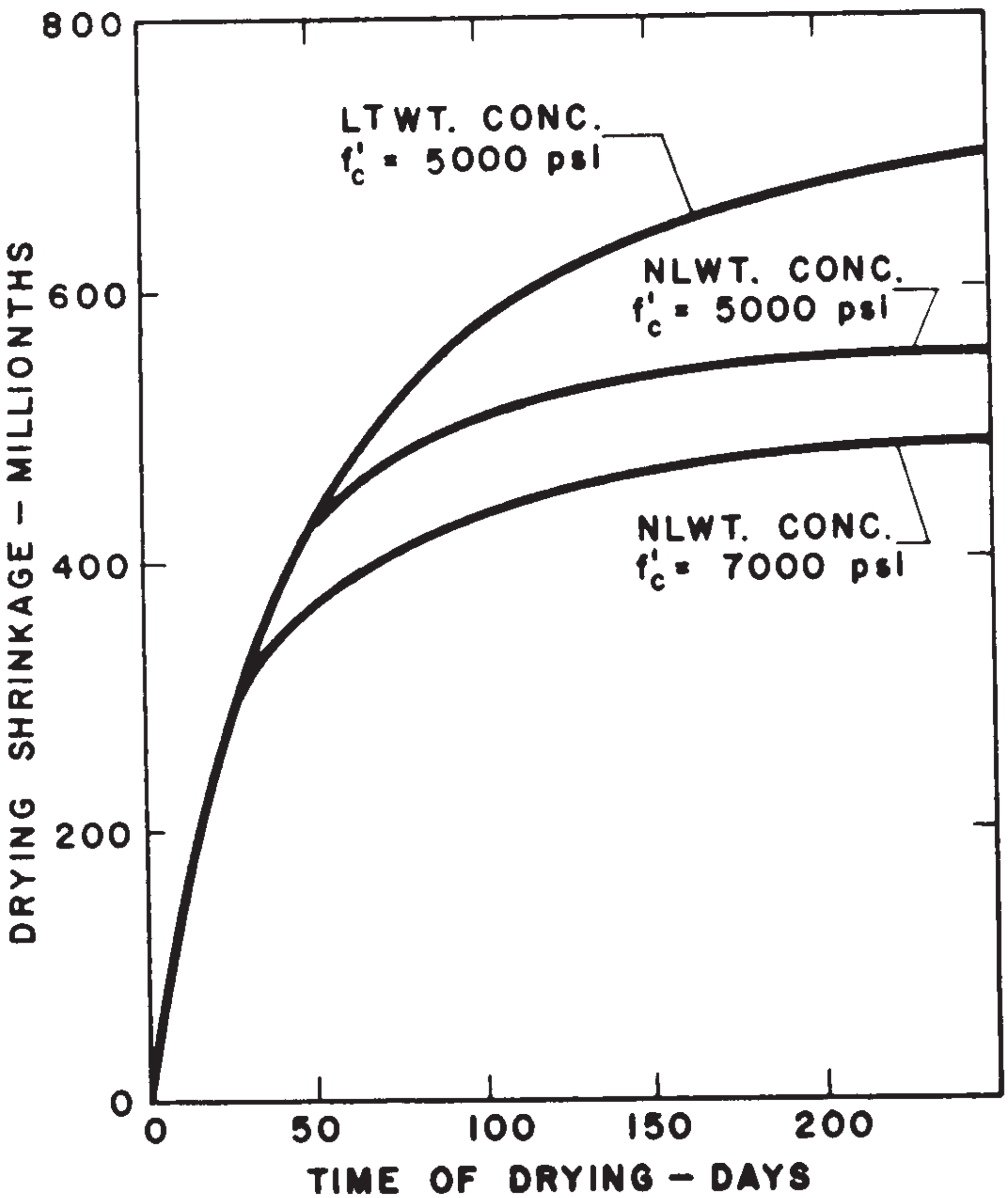
Recent studies by J. A. Hanson[4] reveal appreciable reductions of creep and shrinkage by accelerated curing methods. Atmospheric steam curing, typical of prestressed plants, has been found to reduce creep 20-30% and shrinkage 10-20% for Type I cement; and to reduce creep 30-40%, and shrinkage 25-40% for Type III cement when compared to moist cured samples.

The creep and shrinkage properties for a given concrete are not often pursued specifically by designers who rely rather on arbitrary design values suggested in the literature. However, it often becomes necessary to obtain more exact creep and shrinkage data for a given material -- especially if lightweight aggregates are being considered for an application of prestressed concrete. Typical time and load dependent deformation characteristics for several types of concrete commonly used in prestressed concrete are shown in Figs. 7 and 8.



TYPICAL CREEP CURVES
FOR CONCRETE

FIG. 7



TYPICAL SHRINKAGE CURVES FOR CONCRETE

FIG. 8

These relationships should not be used as representative of all concretes for each strength shown. They are presented merely to illustrate the general creep and shrinkage character of concrete. The magnitude of deformation due to creep and shrinkage may vary considerably, especially with lightweight structural concretes, depending largely upon the aggregate materials. However, the general character of the creep and shrinkage relationship for all concrete is much the same.[5]

Modulus of elasticity, creep and shrinkage are important considerations when calculating cambers and deflections of prestressed concrete members. Such calculations become difficult because there are many factors to be considered:

1. Modulus of elasticity increases with age
2. Creep and shrinkage must be taken into account
3. Reduction in prestress force must be taken into account

The deflection from a given load at the instant of loading is calculated using the modulus of elasticity of concrete at that particular time. For example, at time of release two forces are applied. To determine the initial camber the modulus of elasticity based upon the concrete strength at time of release is used. If no other loads are applied to the member the camber will increase with time as a result of creep and shrinkage.

As a rule for a given stress, creep and shrinkage will produce final strains in concrete 2 to 3 times greater than strains resulting from initial application of the stress. Therefore the modulus of elasticity can be assumed to be 1/2 to 1/3 of the initial modulus of elasticity in deflection calculations. This modulus of elasticity is usually referred to as the reduced modulus of elasticity. Another method employs a factor known as a creep coefficient. This number is also used to reduce the modulus of elasticity. Reasonable values for a creep coefficient are: at 30 days 2.0; at 1 year 2.5. To determine resulting cambers after all creep and shrinkage has taken place, it is necessary to use the reduced modulus of elasticity in place of the initial modulus of elasticity. It would appear that the final camber would be two to three times greater than the initial camber. However, it should be remembered that creep and shrinkage will also reduce the applied prestress force. Thus, camber growth resulting from creep and shrinkage is lessened by the reduction in prestress force.

When additional load is applied to the member, such as the application of dead load, the modulus of elasticity at the time of loading should be used. Long time deflections from dead load are calculated on the basis of the reduced modulus of elasticity. The reduction factor to be used is related to the age of the concrete at the time the

dead load is applied. With live loads of relatively short duration such as snow loads, instantaneous values of modulus of elasticity consistent with the strength of the concrete should be employed in computations.

Tensile Strength

The tensile strength of concrete is a property that is usually ignored in conventionally reinforced concrete with the notable exception of pavements and dams. Consequently it has not received as much attention in the laboratories as other properties. For instance, the tensile stress-strain curve of concrete has not been established, although indirect analyses indicate the relationship is approximately a straight line. Tensile strength becomes important in prestressed concrete. The following equations have been used to relate tensile strength or modulus of rupture to compressive strength:

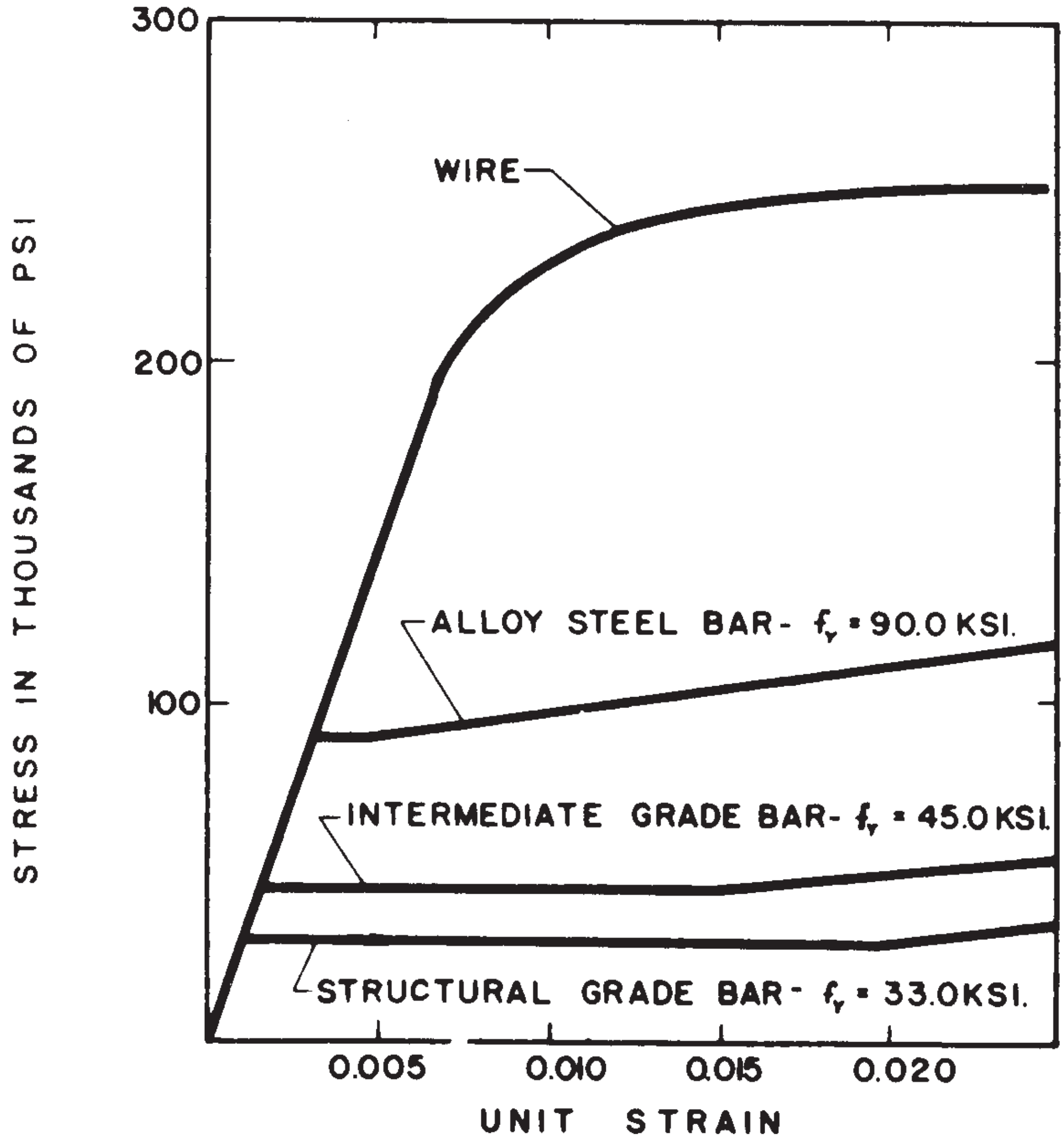
$$f_t = Kf'_c \text{ where K varies from 0.08 to 0.15} \tag{7}$$

$$f_t = 7.5 \sqrt{f'_c} \tag{8}$$

$$f_t = 6 \sqrt{f'_c} \quad \text{(ACI 318-63)} \tag{9}$$

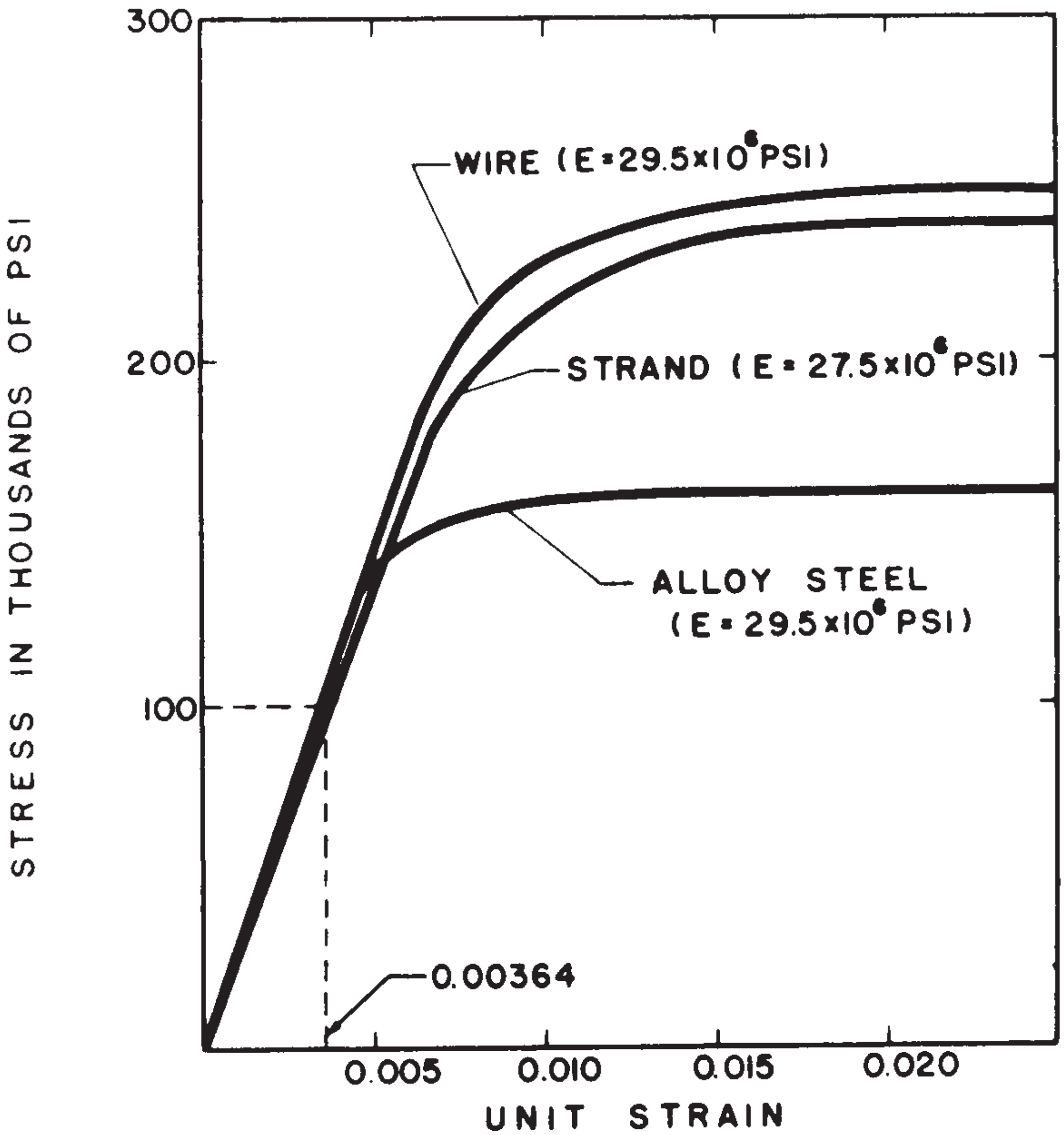
Eq (8) is based upon the best knowledge presently available. It should be used in preference to Eq (7). Eq (9) is recommended in Section 2605 of ACI 318-63 with provisions made via Section 104 of ACI 318-63 to exceed this value provided substantiating evidence is produced.

Tensile strength is often relied upon in working stress design of



TYPICAL STRESS-STRAIN CURVES
FOR
CONCRETE REINFORCEMENT

FIG. 9



TYPICAL STRESS-STRAIN CURVES
FOR
PRESTRESSING STEELS

FIG. 10

basic differences are obvious between the stress-strain relationship of prestressing steels and those of other steels normally used by structural engineers. These are:

1. the high tensile strength of the prestressing steel
2. the absence of a well defined yield point on the high-strength steel curve, and
3. the reduced modulus of elasticity of stranded wire.

The physical properties of steel which are of significance in the design and production of prestressed concrete are:

a. tensile strength
b. one per cent strain under load (ASTM A416 & A421)
c. modulus of elasticity.
d. bond properties

Tensile Strength

Curves in Fig. 10 show stress-strain characteristics of stranded wire which is most widely used for pretensioned prestressed concrete, of straight wire which is used in some pretensioning and a great deal of post-tensioning, and of high tensile strength alloy steel bars which are used in post-tensioned prestressed concrete. These steels all have high tensile strength and show a gradual deviation from linearity with no distinct yield point.

These high strength steels achieve their strength largely through the use of special chemical compositions in conjunction with cold-working. Recently newer high strength strand has been developed with an ultimate strength of 270,000 psi. This produces additional economies by requiring fewer strand.

Yield Strength

As seen in Fig. 9, steels normally encountered in structural design exhibit definite yield points. High strength wire, however, does not and the one per cent extension under load has been selected arbitrarily as the yield strength for design purposes, ASTM A416. This definition of yield strength has supplanted the "0.2% offset" yield strength which was formerly used.

Modulus of Elasticity

Tests on stranded wire yield values of modulus of elasticity which deviate from values obtained from other steels. The reason for this deviation is understandable when the fact is considered that a number of wires twisted together are acting as a unit when they are tensioned. Prestressing strand is made by combining wires from different reels after they have been cold-drawn, and stranding them together on a stranding machine. This group of wires, which has been stranded together is stretched as a unit, and although the modulus of elasticity of the individual pieces of steel is unchanged, the

group which has been twisted together will tend to stretch more than solid steel because the wires tend to untwist. If a strain gage is placed along the axis of a single wire in a strand being tested, the stress-strain properties of that wire are found to be very similar to those obtained from the test on a straight piece of wire. However, if the strain gage is placed over a given length of the strand, the apparent modulus of elasticity resulting from a tensile test will be found to be lower.

This apparent modulus of elasticity of stranded wire is somewhat inconsistent, and may vary as much as one or two per cent within a single reel from any manufacturer. This inconsistency arises from variation in the tightness with which the helical wires are wound together. The variation is even greater from reel to reel. Considering the problems involved, it is rather amazing that a group of 7 wires, twisted together, can yield as consistent stress-strain properties as we actually experience.

The significance of the modulus of elasticity of prestressing tendons is confined primarily to the act of prestressing. This quantity is utilized to establish or check the magnitude of the initial tension placed into the tendons. Neither "E" nor "n", the modular ratio, appears in any of the design expressions for prestressed concrete.

They are indirectly considered in evaluation of the prestress losses arising from elastic, creep and shrinkage shortening of the concrete.

Bond

Bond serves a dual function in pretensioned prestressed concrete. The first of these functions is to transfer load from the steel to the concrete to accomplish the prestressing. This function is termed "prestress transfer bond". The second function is similar to that required for conventional reinforcement, distributing the steel stress to correspond to the magnitude of the change in moment at any cross-section. This is termed "flexural bond".

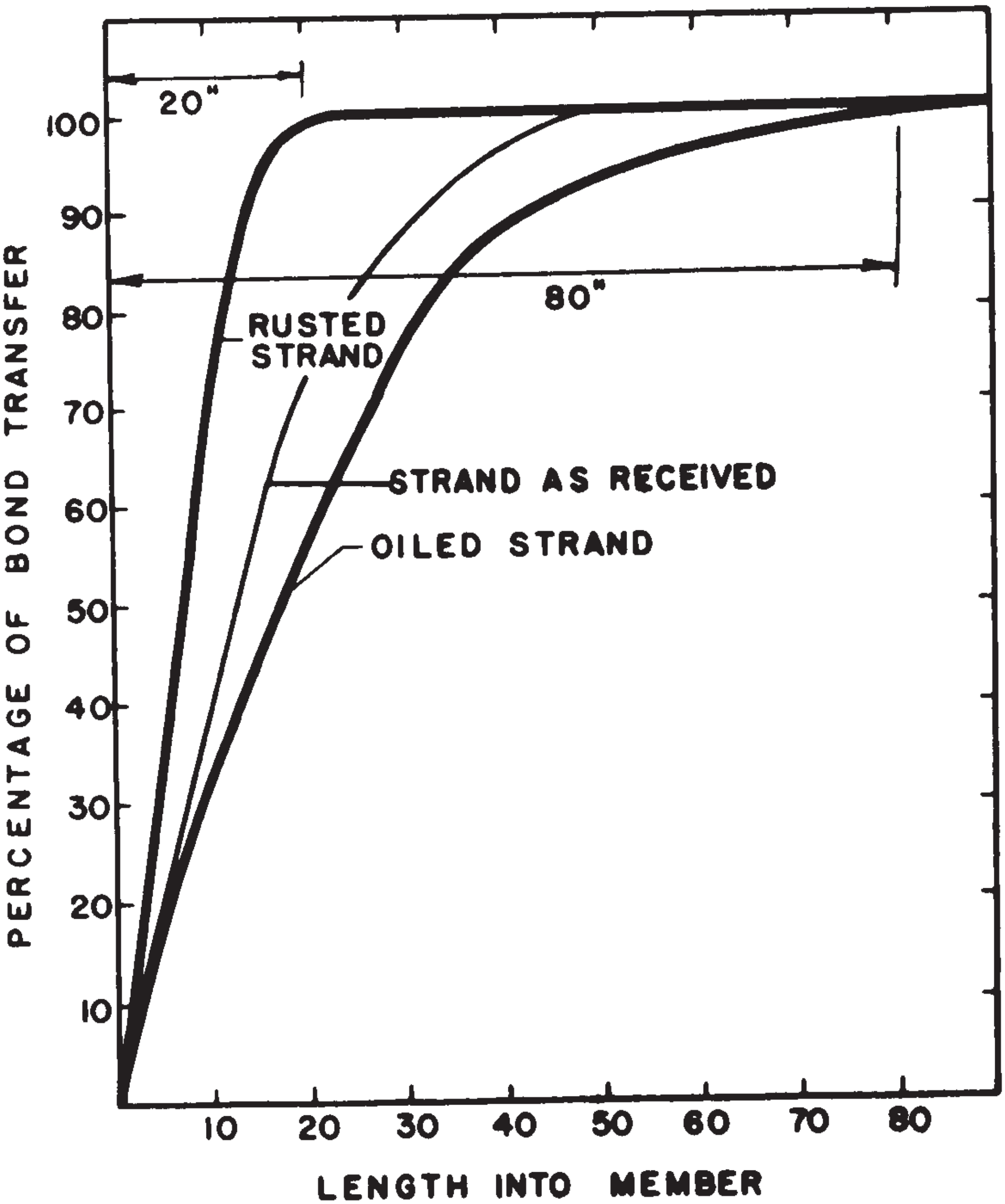
When a pretensioning tendon is stressed, it not only elongates but reduces in diameter. If concrete is placed around the stressed tendon, allowed to harden, and then the stress is released, the steel within the concrete attempts to return to its original unstressed diameter. As the diameter of the tendon tends to increase, a bursting force of great magnitude is applied to the surrounding concrete. This force is so great that a very large frictional force is developed which prevents movement of the steel into the concrete. Thus transfer bond becomes achieved primarily as a consequence of friction rather than adhesion or mechanical anchorage.

The bonding characteristics of conventional reinforcement are a

function of adhesion between the concrete and steel, mechanical anchorage and to a minor degree, friction.

Fig. 11 shows the stress transfer relationship for 7/16 inch diameter prestressing strand which is the same in every respect with regard to tension, size, and concrete embedment with the exception of the surface characteristics of the steel. The values shown on the vertical scale represent the percentage of full stress in the steel at any point, and values shown on the horizontal scale give the distance in from the end of the block of concrete in which the steel is embedded. The curve on the right shows the stress transfer for steel on which a light film of oil was placed, the center curve is the stress transfer for steel as received from the manufacturer, and the left hand curve is a similar representation for steel on which there was a light coat of rust. It may be seen that the full stress from the rusted steel is transferred to the concrete in 20 inches while the steel which was coated with the oil required 80 inches for full transfer.

These representations are shown for comparative purposes only and should not be viewed as absolute values of transfer lengths for the conditions shown. The method of release of prestress force can also significantly affect the length of the transfer zone. An abrupt cutting of strands will probably result in a long zone.



BOND TRANSFER LENGTH
7/16" STRAND

FIG. 11

Gentle slow release will result in a shorter transfer zone. Methods such as cutting the strand by burning can result in variable transfer zone lengths, dependent on the care taken in the burning process. A value of 50 diameters for the transfer zone length is an average value based on a clean strand surface.

Flexural bond stresses are of very low magnitude until after flexural cracks develop. Therefore, flexural bond stresses are insignificant in prestressed concrete through all stages of loading to service loads and become significant only in considering the ultimate flexural capacity. Because the stress in the steel is at high level at the time the precompression is overcome, flexural bond is not called upon to develop the full strength of the steel.

It should be noted that the significance of the transfer zone should be evaluated in terms of the member under consideration. Therefore a long transfer zone may be of no real significance in a long span building member. The expression $(f_{su} - 2/3\ f_{se})D$ (ACI 318-63, Section 2611) for flexural bond is an empirical one based on tests conducted at the Portland Cement Association. When using the equation, investigations can be confined to the sections of the beam, nearest the beam end, that are required to develop their ultimate capacity. In cantilever beams the

section at the support should be investigated.

Loss of Prestress

Because of elastic shortening, creep and shrinkage of concrete and stress relaxation of steel, the initial prestressing force gradually diminishes. This decrease is termed loss of prestress. Loss of prestress is rapid at early ages and gradually reaches a stable condition of effective prestress assumed to be permanent. The magnitude of prestress loss does not significantly affect the ultimate capacity of a member. An error in estimating the loss is reflected in the cracking load and amount of camber.

The ACI-ASCE 323 report suggests that unless specific data representative of the materials and design contemplated are available, steel stress losses may be assumed to be 35,000 psi for pretensioning and 25,000 psi for post-tensioning. Losses due to friction between post-tensioned steel and ducts must be added to the 25,000 psi. This method of estimating losses generally satisfies the requirements of Section 2607 of ACI 318-63.

For lightweight concrete, losses due to elastic shortening, shrinkage, and creep of the concrete should be based on results of tests made with the lightweight aggregates to be used.

FLEXURE

Design for Working Stresses

The tensile strength of concrete is ignored in reinforced concrete design and steel reinforcement is relied upon entirely to resist internal tensile forces. As a result a sizeable volume of concrete is occupying space without being considered as contributing to the load-carrying ability of the flexural member.

The basic principle of prestressing concrete is to induce in some manner permanent compressive stresses into the member before it is loaded. These compressive stresses are concentrated in the regions where tensile stresses are anticipated to occur after the member is loaded. Thus, through this manipulation of internal stresses we are permitted to take advantage of the entire cross-section in the analysis of flexural members to resist working or service loads. Consequently, the net area and moment of inertia of a prestressed member are greater than those quantities in a non-prestressed member of the same dimensions.

In the case of flexural members the compressive stresses imposed to resist eventual tension are created by the application of sufficient force located so that it is most efficient in bringing about the compressive stresses. This is usually accomplished by tensioning steel

elements (wire, stranded wire, or rods). The reaction of the force applied to these elements is eventually taken by the concrete member by bond or anchorages. The prestressing tendons may be pretensioned before the concrete is placed around them and then released to the concrete after it has attained sufficient strength. In this case the loads are transferred from the pretensioned steel to the concrete member by bond. This type of prestressing is called pretensioning. The tendons may also be post-tensioned after the concrete has hardened. Here the forces can be transferred to the concrete one of two ways. One method is solely through bearing devices to which the prestressing tendons are anchored mechanically. Another method is by passing the post-tension tendons through tubes or ducts. After the tendons are stressed they are also anchored mechanically. The ducts are then pumped full of a cement grout thereby providing for transfer of stress by bond.

The flexural analysis of a member prestressed by any method for all stages of loading from fabrication through full design service loads is built around the combined stress formula

$$f = \frac{P}{A} \pm \frac{Mc}{I} \qquad (9)$$

Normally, the use of this formula involves only externally applied direct loads and moments. However, the internal loads and moments created by the prestressing tendons become fundamental features of

the analysis and design of prestressed concrete. The following example is presented to demonstrate the basic principles of design: A simple beam in Fig. 12 A is made of plain concrete having a compressive strength of 5000 psi and is loaded as shown. The bottom fiber stress at mid span under this loading is

$$f_b = -\frac{M_D}{S} - \frac{M_L}{S} \qquad (10)$$

$$= -\frac{w_D L^2/8}{bh^2/6} - \frac{w_L L^2/8}{bh^2/6}$$

$$= -196 - 625 = -821 \text{ psi}$$

This stress would likely exceed the tensile strength of the concrete. If an eccentric load were applied to the ends of the beam so that the resulting stress in the bottom fiber at mid span equals 821 psi or greater, no tension would ever be expected to exist in the concrete at mid span, Fig. 12 B. In order to achieve the greatest compression in the bottom fibers with the least force, F, it must be applied with the greatest eccentricity possible. Allowing 2" to the center of the prestressing steel for concrete cover, the eccentricity becomes 13 inches. Solution of the following expression yields the force required to produce a final bottom fiber stress of zero.

$$0 = \frac{F}{A} + \frac{Fe}{S} - \frac{M_D}{S} - \frac{M_L}{S} \quad \text{(Dead + Live, bottom fiber)} \qquad (11)$$

$$= \frac{F}{300} + \frac{13F}{1500} - \frac{292000}{1500} - \frac{940000}{1500}$$

$$F = 68.5 \text{ Kips}$$

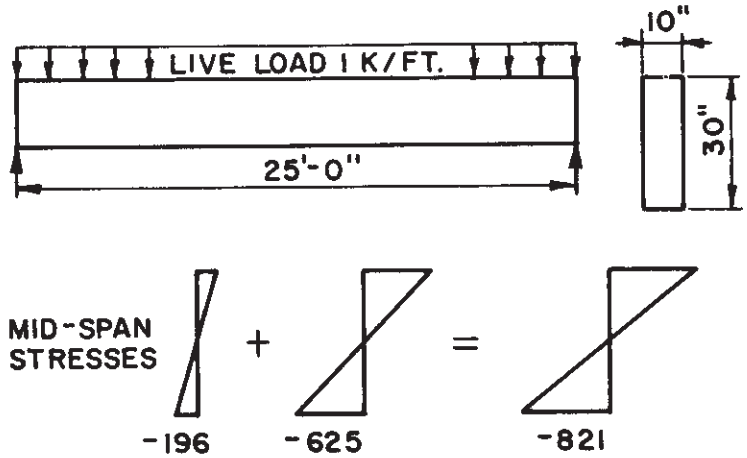


FIG. 12A PLAIN CONCRETE BEAM

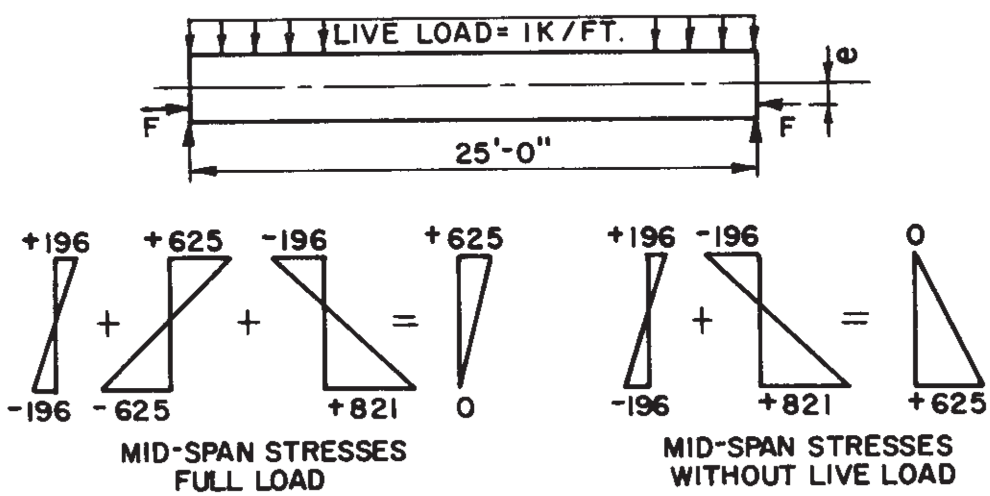


FIG. 12 B PRESTRESSED BEAM-STRAIGHT TENDONS

However, if the live load is not acting, the top fiber stress at mid-span is:

$$f_t = \frac{F}{A} - \frac{Fe}{S} + \frac{M_D}{S} \text{ (Dead only, top fiber)} \qquad (12)$$

$$= -170 \text{ psi}$$

For purposes of this illustrative example only, we are permitting no tensile stresses under any conditions of loading. Therefore, we must reduce the eccentricity and increase the prestress to meet this requirement. Setting $f_t = 0$ in Eq (12) the simultaneous solution of Eq (11) and (12) for F and e yields:

$$F = 94 \text{ Kips}$$

and $e = 8.1$ in.

These values satisfy the requirements at mid-span. At the ends of the beam, however, the flexural stresses arising from the dead load of beam do not exist. Therefore, tensile stresses are present in the top fiber. At the ends the center of the prestressing forces must be within the middle third of a rectangular section or within the kern of any cross-section. These conditions may be satisfied by locating the force in the example beam with an eccentricity of 5 inches plus increasing the prestressing force the additional required amount. They can also be satisfied by deflecting, or draping, down at the center and up at the ends, the stressing elements; or by unbonding some of the elements in the end regions.

Obviously decreasing the eccentricity and increasing the prestressing force does not represent the economical solution.

Tensile stresses are permitted by codes in the top and bottom fibers for design conditions, but for purposes of illustration no tensile stresses were permitted in the above example. Limiting tensile and compressive stresses will be covered in the subsequent material.

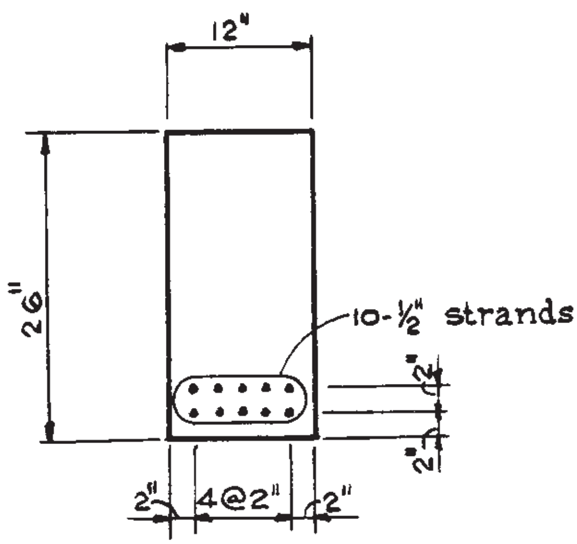
Example: Determine the allowable uniform load that the rectangular beam shown below can carry over a simple span of 36'0".

Design data f'_c = 5000 psi

f'_s = 250,000 psi

Area (1/2" strand) = 0.143 in^2

Allowable bottom fiber tension = - 425 psi



Mid Span Cross - Section

Section Properties

$$A = 26 \times 12 = 312 \text{ in}^2$$

$$S = \frac{12 \times 26^2}{6} = 1352 \text{ in}^3$$

Load Determination

$$f_{bf} = + \frac{F}{A} + \frac{Fe}{S} - \frac{wL^2}{8S} = - 425$$

$$F = 10 \times 0.143 (250,000 \times 0.7 - 35,000)^*$$

$$F = 200 \text{ k}$$

$$- 425 = + \frac{200}{312} + \frac{(200)(10)}{1352} - \frac{w (36)^2 \; 12}{8 (1352)}$$

w_t = 1770 pounds per foot

weight of beam = $\frac{312}{144} \times 150$ = 326 plf

Thus, allowable superimposed load:

$$w_L = 1770 - 326$$

$$= 1444 \text{ plf}$$

ULTIMATE STRENGTH

Determination of the ultimate flexural capacity of prestressed concrete is an important part of design procedure. Even though the designer does not expect the application of load of such magnitude as to cause failure, ultimate strength computations are necessary to insure the proper factor of safety.

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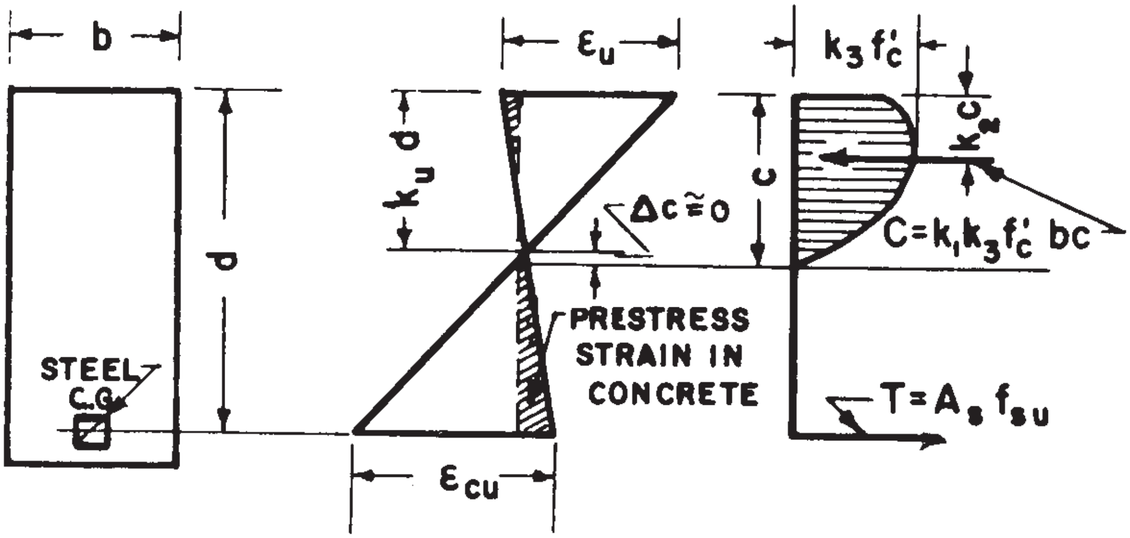
* Stress in prestressing tendons immediately after transfer is $0.70f'_s$ (ACI 318-63). The reduction of 35,000 psi (recommended by ACI-ASCE Joint Committee 323) allows approximately 20% for losses due to elastic shortening, shrinkage and creep of concrete and relaxation of steel stress.

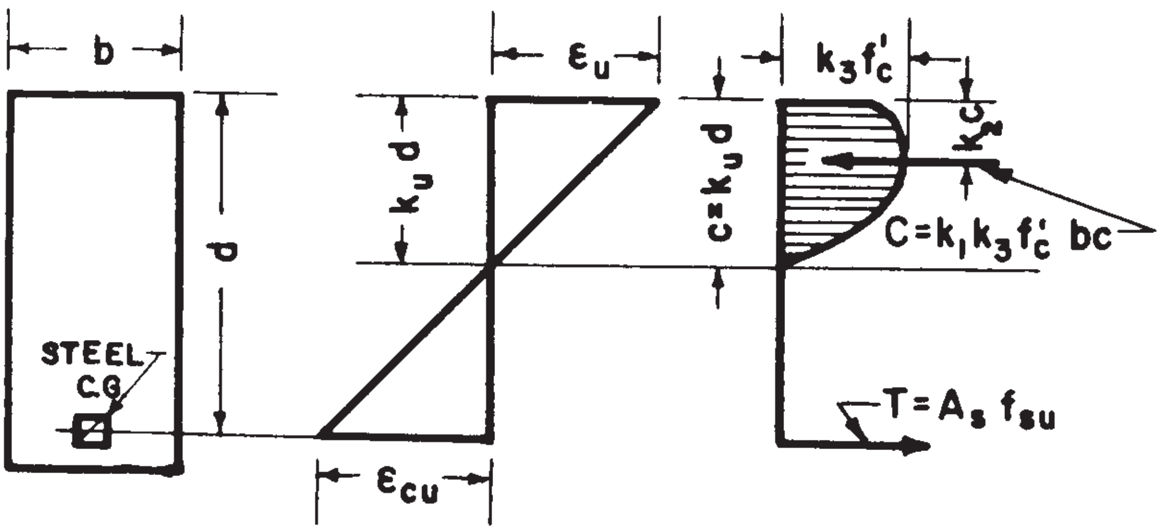
The analysis of prestressed concrete for service load, presented in the foregoing section, presumed complete elasticity of the concrete. Also the analysis is not complicated by the assumption of cracks in the concrete. When one witnesses a test to failure of a prestressed concrete beam it becomes obvious that these assumptions are no longer valid at the time failure occurs. In fact, the behavior of a prestressed member at failure is much the same as that of a conventionally reinforced beam. The primary difference in the computation predicting the ultimate flexural strength of prestressed concrete and reinforced concrete lies in the difference between the stress-strain properties of prestressing steels and conventional reinforcement.

Two sets of equations are involved in the derivation of the expressions predicting ultimate flexural strength of conventionally reinforced as well as prestressed concrete.

One set expresses the conditions of equilibrium which are independent of the type of reinforcement as well as whether it is prestressed or not. The other is based on the assumption that plane sections always remain essentially plane. The conditions of stress and concrete strain at ultimate moment capacity for prestressed concrete and conventionally reinforced concrete are depicted in Fig. 13. The diagrams are shown for both conditions to illustrate that the significance



CONDITIONS AT ULTIMATE MOMENT
PRESTRESSED CONCRETE



CONDITIONS AT ULTIMATE MOMENT
ORDINARY REINFORCED CONCRETE

FIG. 13

of the prestress is rather minor because it influences the strain diagram only slightly. The equations of equilibrium are:

$$T = f_{su}A_s = C = k_1 k_3 f'_c bc \tag{13}$$

and $$M_u = f_{su}A_s\,(d-k_2c) \tag{14}$$

The equation expressing compatibility of strains is:

$$\epsilon_{cu} = \frac{\epsilon_u(1-k_u)}{k_u} \tag{15}$$

Solving Eq (13) for c and substituting into Eq (14) the expression for ultimate moment becomes:

$$M_u = A\ f_{su}d\ \left(1 - \frac{k_2}{k_1k_3} \times \frac{f_{su}}{f'_c} \times \frac{A_s}{bd}\right) \tag{16}$$

Another more convenient form for Eq (16) is:

$$M_u = bd^2f'_cq\left(1 - \frac{k_2}{k_1k_3}q\right) \tag{17}$$

where q is the reinforcement percentage index, $\frac{pf_{su}}{f'_c}$. The problem now reduces to one of evaluating f_{su} and the ratio $\frac{k_2}{k_1k_3}$, ACI 318-63 uses $\frac{k_2}{k_1k_3} = 0.59$.

The ultimate flexural strength formula is presented in ACI 318-63 in a slightly different manner:

1. Rectangular sections, or flanged sections in which the neutral axis lies within the flange:

$$M_u = \Phi\left[A_sf_{su}d\ (1-0.59q)\right] = \Phi\left[A_sf_{su}(d - \frac{a}{2})\right] \tag{26-4}$$

2. Flanged sections in which the neutral axis falls outside the flange:

$$M_u = \Phi\left[A_{sr}f_{su}d\left(1 - \frac{0.59A_{sr}f_{su}}{b'd\,f'_c}\right) + 0.85\,f'_c(b-b')\,t\,(d-0.5t)\right] \quad (26\text{-}5)$$

Where $A_{sr} = A_s - A_{sf}$

and $A_{sf} = 0.85\,f'_c\,(b-b')\,t/f_{su}$

The equations are the same as developed previously except for the addition of the capacity reduction factor Φ. This reduction factor, Φ, is common throughout the code applying to both normal reinforced concrete and prestressed concrete. For ease of calculation the required ultimate moment can be found by dividing by Φ making the determination of the required area of steel and its location straightforward.

For conventionally reinforced concrete f_y may be used for f_{su}. If q does not exceed 0.4, Eq (17) produces predictions for ultimate moment capacity which are in close agreement with test results. The determination of f_{su} for prestressed concrete is not so simple. For the most accurate computations, the stress-strain curve for steel must be used. Assuming well bonded steel a value for ϵ_{cu} is selected from the stress-strain curve and Eq (15) is solved for k_u setting $\epsilon_u = 0.003$. Then Eq (13) is solved for f_{su} substituting k_ud for c and letting $k_1k_3 = 0.67$. A trial and error process must be used until the selected value of ϵ_{cu} agrees with the calculated value of f_{su} on the stress-strain

curve. For most design purposes, however, the expressions presented in the ACI-ASCE Joint Committee 323 Report and the ACI 318-63 Building Code Requirements are satisfactory:

f_{su}	$= f'_s \left(1 - 0.5\,\frac{pf'_s}{f'_c}\right)$	Bonded reinforcement (ACI 318-63 Sec. 2608)
f_{su}	$= f_{se} + 15000$ psi	Unbonded reinforcement (ACI 318-63 Sec. 2608)

Suggested Initial Design Procedures

Contrary to the practice of most designers, the writers feel that the ultimate strength design procedure can be used to good advantage at the outset of a design problem for selecting cross-sections or to determine the feasibility of a proposed cross-section. Other designers prefer to use ultimate strength only as a check on factor of safety. The following examples illustrate the use of the ultimate strength procedure as an initial design tool.

Example: Design a rectangular beam to span 50 feet and to carry a total load, exclusive of its own weight, of 1 kip per lineal foot.

Design data $f'_c = 5000$ psi

$f'_s = 250,000$ psi

Est. beam wt. = 250 lb/ft

Total Design Moment

$M_D = (250)\,(50)^2\,(1.5) = 940,000$ in.-lb.

$M_L = (1000)\,(50)^2\,(1.5) = 3,760,000$ in.-lb.

$\text{Let } d = 2.5\ b$

$\text{Try } q = 0.15$

$M_u = 1.5D + 1.8L = 1.5\ (940,000) + 1.8\ (3,760,000)$ (ACI 318-63)

$M_u = 8,170,000$ in.-lb.

$M_u/\phi = bd^2 f'_c q\ (1 - 0.59q)$ Eq. 17 modified by the capacity reduction factor in ACI 318-63

$M_u/\phi = 8,170,000/0.9 = 9,080,000$ in.-lb.

$= (6.25) b^3\ (5000)\ (0.15)\ (0.91)$

$b = 12\text{-}7/8''$ $d = 31''$ (Beam wt. higher than estimated value)

$h = 35''$

Adjust the moment for the added beam weight of a 13 x 35 in. section and solve Eq (17) for q, $M_u/\phi = 10,450$ in.-k

$q = 0.167$

For purposes of estimation assume $f_{su} = 230,000$ psi.

$$\text{If } q = \frac{230,000p}{5,000} = 0.167$$

$$p = \frac{0.167}{46} = 0.0036$$

$$f_{su} = f'_s\ (1 - 0.5\ \frac{f'_s p}{f'_c})$$

$$= 228,000 \text{ psi}$$

This is close enough for the present purpose

$A_s = (0.0036)\ (31)\ (13)$

$= 1.46\ in^2$ use 10-1/2 in. strands

Once the dimension and steel requirements are established, the

section is analyzed elastically for the various conditions of loading, making use of the appropriate code provisions to set limiting stresses. Some adjustment in the location and number of strands usually results from this analysis. The working stress analysis is omitted here because it is covered in the design examples in the appendix.

Example: Design a rectangular beam spanning 50 feet, for a total load of one kip per foot exclusive of weight of beam. The architect has requested that the beam be no larger than 10 in. x 30 in.

Design data $f'_c = 5000$ psi

$f'_s = 250,000$ psi

$b = 10$ in.

$d = 27$ in.

$$M_L = 3,760,000 \text{ in. lb.}$$

$$M_D = \frac{10\ (30)}{144} \times 150\ (50)^2\ (1.5) = 1,170,000 \text{ in.-lb.}$$

$$M_u = 1.5\ (1,170,000) + 1.8\ (3,760,000) = 8,510,000 \text{ in.-lb.}$$

$$M_u/\phi = 9,480,000 \text{ in.-lb.}$$

$$M_u/\phi = (10)\ (27)^2\ (5000)q\ (1-0.59q)$$

and $q = 0.26 < 0.3$ (ACI 318-63 Sec. 2609)

Est. f_{su} $= 210,000$ psi

$p = \frac{0.26}{42} = 0.0062$

$f_{su} = 250,000\ (1-0.5)\ \frac{(250,000)}{(5,000)}\ (.0062)$

$= 209,000$ psi

And $A_s = (0.0062)\ (10)\ (27)$

$= 1.68\ \text{in.}^2$ use 13-1/2 in. strands

With this high value of $\frac{f_{su}}{f'_c}p$ some difficulty may be encountered in meeting the design stress limitations for the several conditions of loading.

<u>Example:</u> Design a rectangular beam spanning 50 ft., for a load of one kip per foot exclusive of the weight of the beam. Architect's limitation, beam dimension 12 x 25.

Design data $f'_c = 5000$ psi

$f'_s = 250,000$ psi

$b = 12$ in.

$d = 21$ in.

$M_u = 8,850,000$ in. lb.

$= (12)\ (21)^2\ (5000)q\ (1-0.59q)$

$q = 0.47 > 0.3$ (section unsatisfactory)

The beam must be increased in dimension or possibly a composite T-beam using a

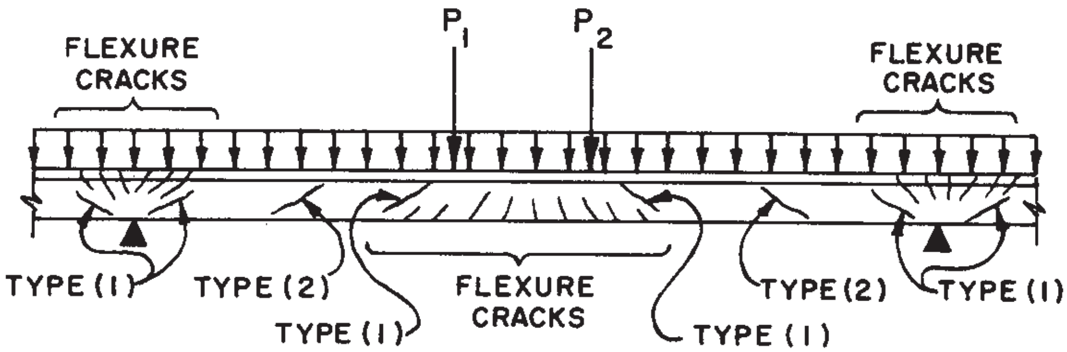
cast-in-place concrete topping might
satisfy the conditions.

SHEAR

Extensive research work during the past fifteen years has led to a basic understanding of the phenomenon of shear in conventionally reinforced and prestressed concrete flexural members. The provisions for shear in the ACI Building Code (ACI 318-63) rest upon this foundation of research with some attempt to simplify equations to ease the task of the designer.

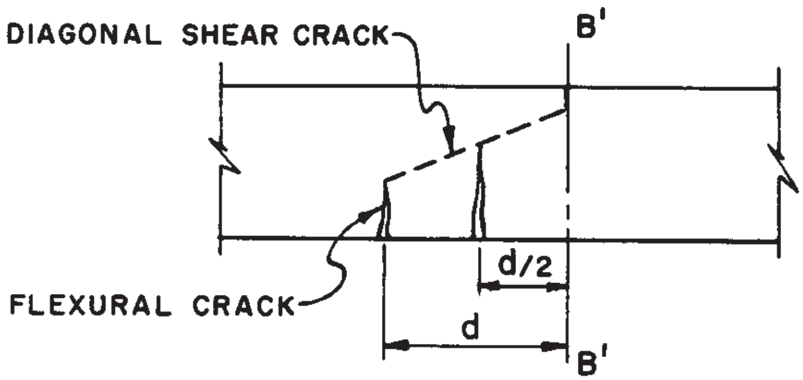
Probably the most important finding by researchers is the fact that the phenomenon of shear is very complicated and involves many variables. Consequently, simplified design expressions, though possible, run the risk of being overly conservative in some cases or possibly too liberal in other cases.

The design expressions of the ACI Building Code are in many respects too cumbersome for general design usage. However, nothing forbids designers from using conservative, simplified equations with the knowledge that in some cases they may be overly conservative, falling back upon the more complicated equations when a detailed analysis may prove to be more economical.



TYPES OF DIAGONAL CRACKS OCCURING
IN A PRESTRESSED CONCRETE MEMBER

FIG. 14



THEORY FOR EQUATION (26-12) FOR V_{ci}

FIG. 15

The diagonal crack that forms in the shear areas of a concrete flexural member results from the weakness of concrete in tension. Research studies show there are two types of diagonal cracks. Type 1, see Fig. 14, is generally called the flexural-shear crack; Type 2 is called the shear crack (or diagonal tension crack).

The shear crack occurs in regions of high shear and low moment. They originate near the centroid of the cross-section, progressing upward and downward diagonally till failure occurs. The mode of formation of the shear crack immediately suggests the use of principal stress equations to determine the principal tensile stress at the centroid of the cross-section. The horizontal and vertical stresses at the centroid result from the prestress force and its vertical component, if strands are draped, and the shear force. The principal stress is expressed as:

$$f_t = -\frac{f_{pc} + f_v}{2} + \sqrt{\left(\frac{f_{pc} + f_v}{2}\right)^2 + v_{cw}}$$

where $f_t \leqq 4\sqrt{f'_c}$

$$f_{pc} = \frac{F}{A_c}$$

$$f_v = \frac{F_p}{A_c}$$

$$v_{cw} = \frac{V_{cw}Q}{Ib}$$

Because this expression is cumbersome for the designer, the ACI

Building Code offers an alternate solution:

$$V_{cw} = b'd\,(3.5\ \sqrt{f'_c} + 0.3\, f_{pc}) + V_p \qquad (26\text{-}13)$$

Flexural-shear cracks occur in regions of moderate flexure and moderate shear. They originate as vertical flexural cracks at the extreme tensile fiber. As they progress upward with increasing load, they bend over diagonally up to and sometimes through the compression zone. Again the mode of formation immediately suggests a means of solution, that is, if the formation of the flexural crack is prevented, the formation of the flexural-shear crack is prevented. Thus, at any cross-section -

$$M_{cr} \leqq \frac{I}{y} \quad (6\ \sqrt{f'_c} + f_{pe})$$

$$\text{where } f_{pc} = \frac{F}{A} \pm \frac{Fey}{I}$$

The corresponding shear is directly related to the M/V ratio of the cross-section, or

$$V_{cr} = \frac{M_{cr}}{M/V} = \frac{I}{y}\,(6\ \sqrt{f'_c} + f_{pe})\,\frac{1}{M/V}$$

However, research has indicated that even though flexural cracks occur at a section, the section can still resist nominal shear in addition to flexural considerations. Therefore,

$$V_{ci} = V_{cr} + 0.6\, b'd\sqrt{f'_c}$$

With minor exceptions this expression for V_{ci} and the corresponding expression for M_{cr} are identical to those appearing in ACI 318-63.

$$V_{ci} = 0.6\ b'd\ \sqrt{f'_c} + \frac{M_{cr}}{M/V-d/2} + V_d \geqq 1.7\ b'd\sqrt{f'_c}$$

$$\text{where } M_{cr} = \frac{I}{y}\ (6\sqrt{f'_c} + f_{pe} - f_d)$$

The term d/2 was added by the Code Committee on the basis that the critical shear section is located a distance d/2 from the section where the flexural crack originates. (See Fig. 15.) The terms f_d and V_d refer to stress due to dead load and shear due to dead load respectively.

Because of the conservative nature of shear equations in general, the refinements realized by the addition of d/2, f and V are not significant and they are an additional burden to design calculations.

It is necessary to point out that the M_{cr} equation is rational in every respect. However, the step from the M_{cr} equation to the V_{ci} equation is empirical based upon research data. Whereas the V_{cw} equation is a simplification intended to replace the principal stress expression, the V_{ci} equation is partially rational and partially empirical. V_{cw} and V_{ci} expressions are completely independent and are related to two different regions of the member with different stress conditions.

The shear crack, V_{cw}, governs in regions of maximum shear and minimum flexure such as near the supports in a simple beam or near the points of contraflexure in a continuous beam. The flexural-

shear crack, V_{ci}, governs in regions where both shear and flexure are acting simultaneously such as near concentrated loads, near the supports of continuous beams, or between the supports and centerline of uniformly loaded simple beams.

Most prestressed flexural members are uniformly-loaded simply-supported beams. For this type of member the critical flexural-shear section is neither in regions of maximum flexure nor in regions of maximum shear, but lies, within tolerable limits between 0.2L and 0.3L. For design purposes the quarter-point can be assumed to be the critical section without serious error.

For the particular case of a uniformly-loaded simply-supported beam, flexural-shear (V_{ci}) at the quarter-point can be conservatively estimated by the equation

$$V_{ci} = 0.6\, b'd\sqrt{f'_c} + 0.25\, W_d L + 0.40 W_l L^*$$

where W_d = dead load in pounds per foot
W_l = live load in pounds per foot
L = length of beam in feet

Shear reinforcement is designed to carry excess shear by equation 26-10 of ACI 318-63:

$$A_v = \frac{(V_u - \phi V_c)\, s}{\phi f_y d}$$

Also ACI 318-63 specifies minimum shear reinforcement by the equation 26-11:

*This equation was derived from Eq (26-12) with conservative assumptions by R. C. Elstner.

$$A_v = \frac{A_s}{80} \cdot \frac{f'_s}{f_y} \cdot \frac{s}{d} \sqrt{\frac{d}{b'}}$$

The procedures for shear reinforcement design are essentially the same as those used for conventionally reinforced concrete (Chapter 17 of ACI 318-63) except for provisions pertaining to minimum shear reinforcement. The following equation expresses the minimum web reinforcement prescribed by Par. 1706(b)

$$A_v = 0.0015\, b's$$

Comparison of this equation and Eq 26-11 shows they are very different. In conventionally reinforced concrete the minimum shear steel requirements only become effective when the shear to be carried by web reinforcement is relatively low. On the other hand, Eq 26-11 is effective over a wide range of($V_u - \phi V_c$). Furthermore whereas shear reinforcement can be omitted in conventionally reinforced concrete, Eq 26-11 requires a specified minimum shear reinforcement in all prestressed beams.

ACI 318-63 does permit omission of shear reinforcement in prestressed beams if "it is shown by tests that the required ultimate flexural and shear capacity can be developed when web reinforcement is omitted" (Par. 2610(c).

Extensive mathematical investigations of simply-supported uniformly-loaded beams by the Portland Cement Association show that Eq 26-10

supersedes Eq 26-11 only in extreme cases, and furthermore that Eq 26-13 (V_{cw}) is rarely critical in shear design except in I-beams.

Tables 1 through 3 are summaries from the PCA investigations[6] and give those values of h/L and A_s/A at which the required web reinforcement just equals the minimum amount specified by Equation 26-11 of the Code. The tables also give the corresponding superimposed live load, w (lb. /ft.) which the member can carry at the quoted parameters. If the member is loaded heavier than with this load, web reinforcement beyond the specified minimum will be required, in accordance with Equations 25-12 and/or Equation 26-13 of ACI 318-63.

It should be noted that the quoted figures are averages taken from extensive load tables[6]. They, hence, can serve only as a guide with about 10 per cent maximum deviations from the exact values.

Table 1

Single tees; 6 ft. wide; 8-3/4 in. web; height: 24 in. to 36 in. Strands draped at mid-span and anchored near CGC.

	$100A_s/A$	0.3	0.5	0.7
No topping	w (lb. /ft.)	550	650	950
	h/L	0.045	0.037	0.045
2 in. topping	w (lb. /ft.)	650	700	2600
	h/L	0.050	0.043	0.074

Table 2

Double tees; 4 ft. wide; 2 in. flange; 4-1/2 in. web (at flange height, with taper 1:12) 10 to 14 inches; parallel strands.

	$100A_s/A$	0.3	0.5	0.7
No topping	w (lb. ft.)	1200	1500	1650
	h/L	0.059	0.065	0.069
2 in. topping	w (lb. ft.)	900	1650	>2500
	h/L	0.051	0.065	>0.09

Table 3

Double tees; 5 ft. wide; 6 in. web (at flange); height: 14 to 18 inches; parallel strands; 2 in. flange.

	$100A_s/A$	0.3	0.5	0.7
No topping	w(lb. /ft.)	1700	2000	2000
	h/L	0.065	0.069	0.068
2 in. topping	w(lb. ft.)	1200	2100	> 2000
	h/L	0.055	0.067	>0.09

Inverted Tees and Ledger Beams

A large variety of inverted tees and ledger beams, varying from 20 in. to 40 in. in height and from 6 in. to 20 in. in width, was investigated. It was found that the critical h/L values were always above 0.10, except for strands draped at mid-span with end anchorages near the CGC. In that case, the critical h/L ratio could

drop to 0.07. This is still unusually high for prestressed concrete construction, so that minimum web reinforcement will practically always suffice for these types.

The following is a sample problem indicating the use of the previously given tables.

PROBLEM A 6 ft. wide single tee (h = 30 in.) floor member is to carry a live load of 175 lb/sq. ft. on a span of 50 ft. The strands shall be draped at mid-span and anchored near the CGC. A 2 in. topping is to be considered in composite action. Does minimum web reinforcement suffice in this case?

ANSWER

The h/L ratio is 30/50 x 12 = 0.05. The load is w = 6 x 180 = 1080 lb/ft. The flexural analysis will show that this load can be carried with less than 0.5% prestressing steel. The shear capacity according to Table 1, however, is only about 700 lb/ft. for this reinforcing ratio. One, therefore, will have to do one of the

following things in order to avoid extra stirrups:

a) Shift the drape point towards the third or quarterpoint or:

b) Lower the end anchorages as much as possible stresswise, or:

c) Increase the prestressing steel to slightly more than 0.5%.

Although examples of design problems contained within this publication show shear calculations rigidly following ACI 318-63, the following tabulation serves to establish the critical sections of flexural members requiring examination for shear. At many of these points the designer has the option of using a rigorous application of code equations, the PCA summaries when applicable, or Elstner's modification of (26-12).

Table 4 -- Critical Sections For Shear

I	Simple Beam -- Concentrated Load		
		V_{ci} at distance from support, d.	Use (26-12)*
		V_{cw} at support	Use (26-13)
II	Simple Beam -- Uniform Load		
		V_{ci} at quarterpoint	Use (26-12) or Elstner modification
		V_{cw} at support	Use(26-13)

* M/V = the longest length in inches from a support to the load point minus d.

II Simple Beam -- Uniform Load (continued)

OR

	V_{ci} and V_{cw}	Use PCA Summaries
III	Continuous Beam -- Concentrated Load	
	V_{ci} at distance d from support	Use (26-12)
	V_{cw} at point of contraflexure	Use (26-13)
	V_{ci} at support	Use (26-12)
IV	Continuous Beam -- Uniform Load	
	V_{ci} at quarterpoint between points of contraflexure	Use (26-12) or Elstner's Modification*
	V_{cw} at point of contraflexure	Use (26-13)
	OR	
	V_{ci} and V_{cw} above	Use PCA Summaries*
	V_{ci} at supports	Use (26-13)
V	Cantilevers -- Uniform or Concentrated Load	
	V_{ci} at support	Use (26-12)**

---

*Using L in equations to be distance between points of contraflexure.

** $\frac{M}{V} = \frac{1}{2}$ L in inches.

The draping of the strands is important to the shear capacity. V_{ci} can be increased by keeping the strands close to the tension face throughout the member even though flexural requirements might allow more concentric prestressing. In the case of draped strands both V_{ci} and V_{cw} reach an optimum with strands draped near the quarterpoint, thereby decreasing the amount of shear steel.

TOLERANCES

The entire subject of tolerance in prestressed concrete, or in any material for that matter, becomes a matter of compromise. Tight tolerances must be established to satisfy architects' and engineers' requirements. At the same time, tolerances must be established that are attainable at a reasonable cost. The following tolerances were developed by the Workmanship Committee of the Prestressed Concrete Institute. They are considered as being typical for a wide variety of prestressed concrete building sections. They are intended for use as a guide to indicate a reasonable standard of performance. If specific job conditions require closer tolerances, it is recommended that the producers of prestressed concrete in the area be consulted.

Dimensional Tolerances

1. Cross-sectional dimensions

    sections less than 6 inches $\pm$ 1/8"

6 inches to 18 inches ± 3/16"

18 inches to 36 inches ± 1/4"

over 36 inches ± 3/8"

2. Overall length ± 1/8" per 10 ft.

   Maximum limitations ± 3/4"

3. Deviation from straight line

   (sweep) 1/8" per 10 ft.

4. Deviation from specified design camber after installation, assuming a design producing minimum camber

   ± 1/8" per 10 ft. of length

5. Differential camber between adjacent units after installation, assuming sections of similar length and cross-section, to be limited to a maximum of one-half the allowance for deviation from design camber in 4 above.

CONNECTIONS

Connections between prestressed members can be accomplished in a variety of ways. The more typical methods are: welding, gravity, cast-in-place concrete, and post-tensioning.

Gravity connections are, of course, the simplest. This connection usually consists of a precast prestressed member resting on a supporting member. Flexible bearing pads can be used to provide

even load distribution, provide for joint rotation, and are inexpensive insurance against adverse stress conditions occurring in the bearing area.

Cast-in-place connections between precast prestressed members have the appearance and behavior of monolithic concrete. They can be constructed and designed to distribute live loads or both superimposed dead loads and live loads.

Welded connections can be quickly completed in essentially any weather. If heavy dead loads are to be placed on the beam, make tack welds during erection and provide full fillet welds after all dead loads are in place, thus reducing stresses in the welds. Since prestressed beams usually tend to shorten slightly over a period of time, and if the supporting frame is inflexible, one end of each beam should be free to slide.

Post-tensioned connections are good for the resistance of high moments. All post-tensioning anchorage and devices should be installed in accordance with manufacturers' recommendations. The conduits containing the tendons should be grouted. When unusual conditions dictate, exposed tendons may be used if deterioration of the steel is prevented by some specific means.

When prestressed beams, tees, or other members are rigidly attached

to supporting columns or girders, the effects of future creep, shrinkage, and temperature should be considered. Shortening in the members due to these factors will create tensile stresses in a member if its ends are rigidly held down. The solution to this problem, however, can be approached in several ways. The rate of both creep and shrinkage is rapid when the members are newly cast, but drops off rapidly with time. Therefore, these effects can be minimized by providing for an amount of time, either before placing the members in the structure or before attaching both ends of the member to the supporting structure.

In any detail, the bars which are attached to the steel bearing plates or angles must be designed to resist tensile forces. If, however, one end of the member is allowed to move and is not rigidly tied down, the problems due to future creep, shrinkage and thermal changes are virtually eliminated; but in this case, stability, where lateral loads must be resisted, should be investigated. In single bay structures, this problem seldom develops, because the supporting walls or columns are usually limber enough to bend slightly to relieve the tension forces.

ACI-ASCE Committee 512 has completed a report[7] that suggests methods by which joints and connections for use in precast concrete construction may be designed.

The Prestressed Concrete Institute has published a report[8] by the PCI Committee on Connection Details which is a collection of details, schematically represented, of types of connections which have proven successful under field conditions.

PARTIAL PRESTRESSING

This is a method of design that consists mainly of designing the prestressed member for behavior rather than for limited tensile stresses. The necessary steel is provided to take ultimate loads. However, the strands either are not stressed the normal amount or some of them are stressed only to a nominal tension sufficient to keep the strand straight. Another method used is to provide some of the necessary steel in the form of mild reinforcing rods rather than strand.

With this method of design, the behavior of the member mainly, as pertaining to camber and deflection, can more easily be controlled and predicted. Dr. P. W. Abeles, English consulting engineer, suggests a permissible concrete tensile stress of 750 psi under working load as a reasonable figure to use in designing by this method.

SPECIAL DESIGN CONSIDERATIONS

This publication makes no attempt to cover such special design considerations as occur in areas subject to hurricanes, tornadoes,

earthquakes or other exceptional conditions which require special engineering attention. It is recommended that the services of consultants particularly qualified to handle these specialized designs be obtained for these problem areas.

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7. ACI-ASCE Committee 512, "Suggested Design of Joints and Connections in Precast Structural Concrete", Proceedings of the American Concrete Institute, V. 61, No. 51, ACI Journal, August 1964.

8. PCI Committee on Connection Details, Connection Details for Precast-Prestressed Concrete Buildings, Prestressed Concrete Institute, 1963.

NOTATIONS

A	=	Gross cross-sectional area of concrete
A_s	=	Cross-sectional area of reinforcement
A_{sf}	=	Area of reinforcement to develop overhanging flanges
A_{sr}	=	Area of reinforcement to develop web
A_v	=	Area of web reinforcement
b	=	Width of compression face of flexural member
b'	=	Average width of web of flanged member
c	=	Distance from neutral axis to extreme fiber
CGS	=	Centroid of prestressing steel
CGC	=	Centroid of concrete section
d	=	Depth from extreme compressive fiber to CGS
e	=	Eccentricity of applied prestressing with respect to neutral axis
E_c	=	Modulus of elasticity of concrete.
F	=	Applied prestressing force
f	=	Stress in concrete
f'_c	=	Compressive strength of concrete
f'_{ci}	=	Compressive strength of concrete at time of initial prestress
f'_s	=	Ultimate tensile strength of steel
f_{se}	=	Effective steel prestress after losses
f_{su}	=	Stress in prestressing steel at ultimate load

f_y = Yield strength of reinforcement

h = Total depth of beam

I = Moment of inertia of gross concrete section

k = With various subscripts denotes coefficients describing stress block at ultimate flexural capacity

M = Applied moment

M_u = Ultimate moment capacity

p = A_s/bd - steel ratio

q = $f_{su}p/f'_c$ -reinforcement percentage index

S = Section modulus

s = Spacing of web reinforcement

V_c = Shear carried by concrete

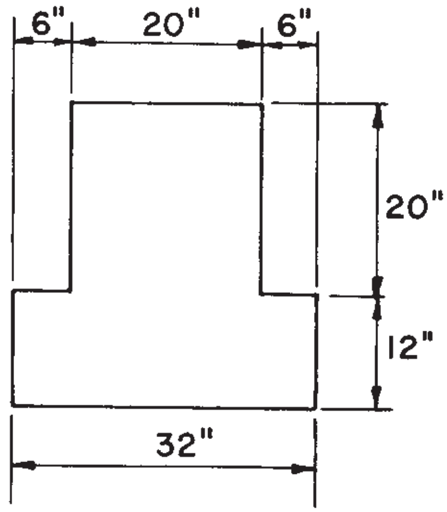
V_{ci} = Shear at diagonal cracking due to all loads, when cracking is the result of combined shear and moment

V_{cw} = Shear force at diagonal cracking due to all loads, when cracking is the result of excessive principal tension stresses in the web

V_u = Shear due to specified ultimate load

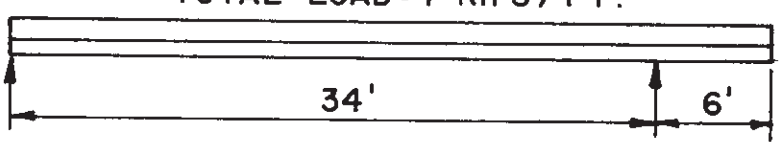
ϵ_c = Strain in concrete

ϕ = Capacity reduction factor (ACI 318-63)



DESIGN ABOVE LEDGER BEAM FOR THE FOLLOWING CONDITIONS:

TOTAL LOAD = 7 KIPS / FT.



LOADS

D.L.	GIRDER :	820 #/FT.
	ROOF SLABS, ROOF., INSUL.	3780
	TOTAL D.L.	4600 #/FT.
L.L.		2400
		7000 #/FT.

PROPERTIES (ORIGINAL ASSUMPTIONS)

f'_c = 5000 P.S.I.

f'_{ci} = 4000 P.S.I.

f'_s = 250,000 P.S.I.

NORMAL WT. CONCRETE

SECTION PROPERTIES

	A	y	Ay	d	Ad^2	I_o
20 x 32	640	16	10,240	1.8	2,075	$20 \times 32^3/12$ = 54,615
2 x 6 x 12	144	6	864	8.2	9,685	$12 \times 12^3/12$ = 1,730
	784		11,104		11,760	56,345

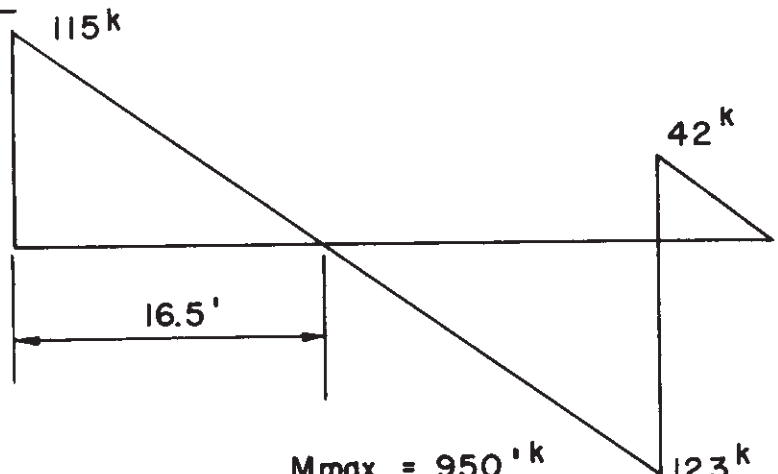
$$\bar{y} = \frac{11{,}104}{784} = 14.2'' \qquad y_t = 17.8''$$

$$I = 11{,}760 + 56{,}345 = 68{,}105 \text{ in.}^4$$

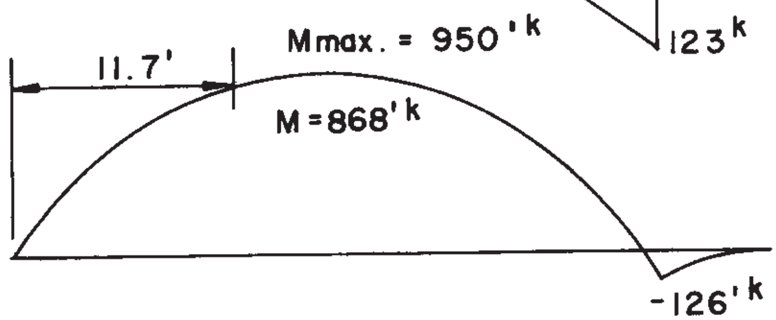
$$Sb = \frac{68{,}105}{14.2} = 4800 \text{ in.}^3$$

$$St = \frac{68{,}105}{17.8} = 3830 \text{ in.}^3$$

SHEAR DIAGRAM



MOMENT DIAGRAM



ESTIMATE FINAL PRESTRESSING FORCE

ALLOWABLE TENSILE STRESS IN PRECOMPRESSED TENSILE ZONE = $6\sqrt{f'_c} = 6\sqrt{5000} = 425$ P.S.I.

$$-425 = +\frac{F_f}{A} + \frac{F_f e}{S_b} - \frac{M_T}{S_b}$$

WHERE:

F_f = TOTAL FINAL PRESTRESS FORCE AFTER ALL LOSSES

A = AREA OF UNCRACKED SECTION

M_T = MAXIMUM TOTAL MOMENT, (DL + LL)

e = ECCENTRICITY OF APPLICATION OF PRESTRESS FORCE

ASSUME $e = 14.2 - 3.0 = 11.2''$

S_b = BOTTOM FIBER SECTION MODULUS

TRY 29 $\frac{1}{2}''$ DIA. 7 WIRE STRANDS ULT. STRENGTH = 250,000 P.S.I.

AREA OF $\frac{1}{2}''$ STRAND = 0.144 in^2

INITIAL TENSIONING STRESS = .70 (250,000) = 175,000 P.S.I.

F_o = 29 (.144) (175,000) = 730 KIPS (FORCE AT JACKING)

F_f = (175,000 - 35,000) (29) (.144) = 585 KIPS.

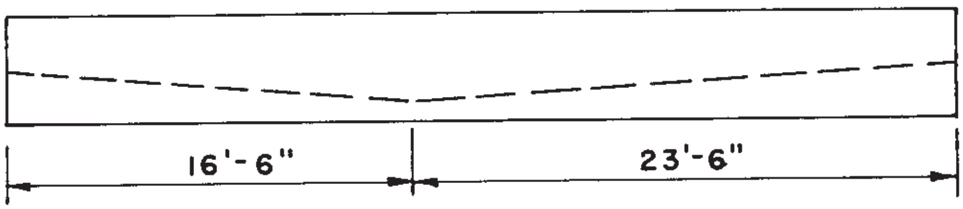
SECTION 2607 (a) OF THE CODE DOES NOT SPELL OUT THE VALUES FOR PRESTRESS LOSS.

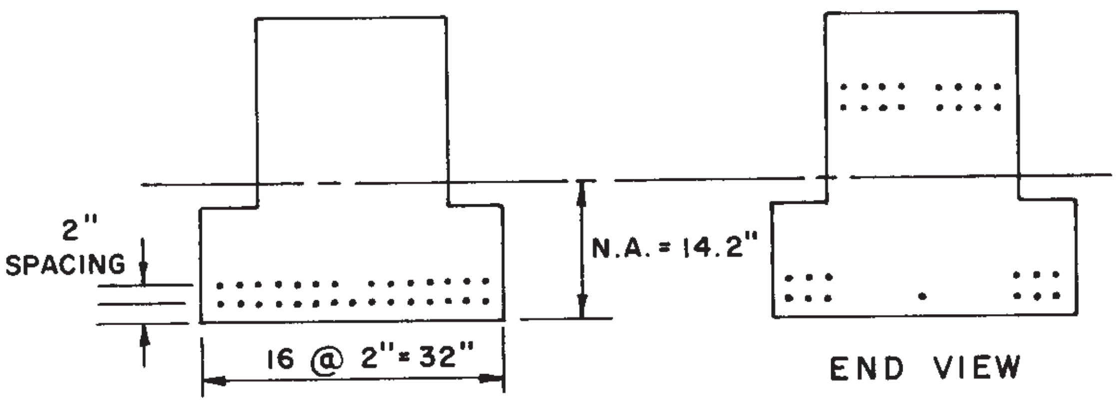
THE TENTATIVE RECOMMENDATIONS SUGGESTED THE 35,000 P.S.I. USED ABOVE FOR PRETENSIONING.

THIS REPRESENTS A 20% LOSS FOR 250 K STRAND AND 18.5 % FOR 270 K STRAND. LIGHTWEIGHT CONCRETE LOSSES WILL BE HIGHER THAN 35,000 P.S.I.

STRAND PATTERN

TRY SINGLE POINT DEPRESSION





SECTION A-A

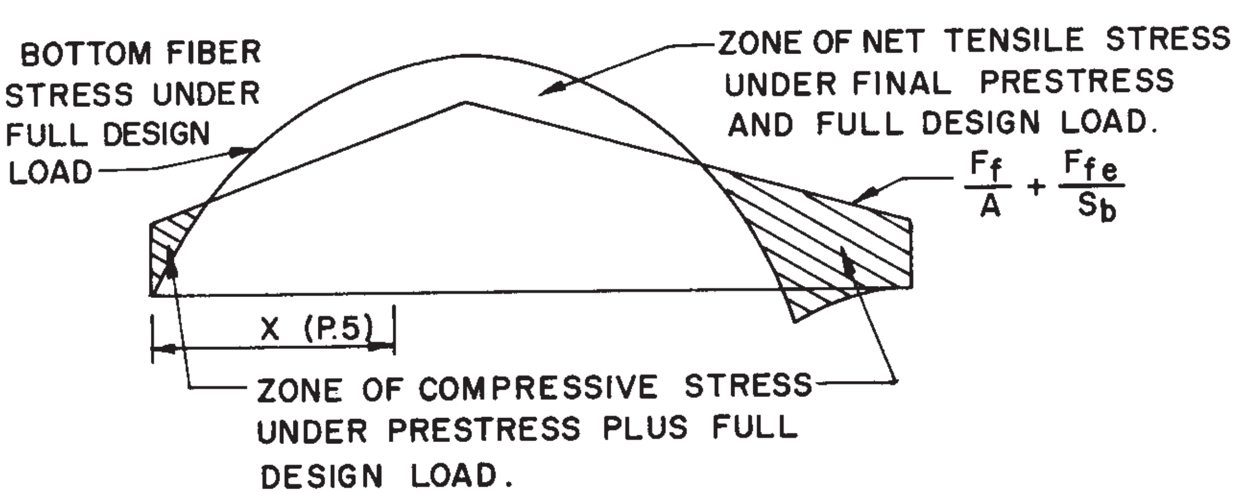
$e = 11.2"$

END VIEW

$$\bar{y} = \frac{7(2) + 6(4) + 16(23)}{29} = 14"$$

$e = .2"$ SAY 0

DETERMINE POINT OF MAXIMUM TENSILE STRESS



IF f_b = NET TENSILE STRESS IN BOTTOM FIBER

THEN
$$f_b = \frac{-12\left[115x - \frac{7x^2}{2}\right]}{S_b} + \frac{F_f}{A} + \frac{F_f(11.2)}{S_b} \cdot \frac{x}{16.5}$$

FOR f_b MAX., $\frac{dfb}{dx} = 0$

$$\frac{dfb}{dx} = -12\left[\frac{115-7x}{S_b}\right] + \frac{11.2\,F_f}{16.5\,S_b} = 0$$

$$84x = 1380 + .68\,(585)$$

$$x = \frac{983}{84} = 11.7'$$

@ $x = 11.7'$ $\quad e = \frac{11.7}{16.5}(11.2) = 7.9$ in.

FOR SIMPLE SPANS WITH CENTER POINT DEFLECTED STRANDS AND UNIFORM LOADING, THE POINT OF MAXIMUM NET TENSILE STRESS WILL BE NEAR THE $\frac{3}{8}$ PT. OF THE SPAN. IN THIS EXAMPLE WE COULD HAVE USED THIS APPROXIMATION (12.7') AND HAVE BEEN SUFFICIENTLY CLOSE. IT IS ALSO POSSIBLE TO FIND THE MAXIMUM STRESS GRAPHICALLY. NOTE THAT THE ONLY TIME THE POINT OF MAXIMUM STRESS IS AT THE POINT OF MAXIMUM MOMENT IS WHEN STRAIGHT STRANDS ARE USED.

$$f_b\ \text{MAX.} = +\frac{585}{.784} + \frac{11.7}{16.5}(11.2)\frac{585}{4.8} - \frac{12\left[115 \cdot 11.7 - \frac{7(11.7)^2}{2}\right]}{4.8}$$

$$= +747 + 970 - 2170$$

$$= -453\ \text{P.S.I.} > 425\ \text{P.S.I.} = 6\sqrt{f'_c}$$

IF $f'_c = 6000$ P.S.I. $\quad f_b\,(\text{ALLOW.}) = 6\sqrt{6000} = 465$ P.S.I.

CHECK ULTIMATE MOMENT AT SAME POINT

$$M_u = 1.5D + 1.8L$$

$$= 1.5 \times \frac{4.6}{7} \times 10{,}400 + 1.8 \times \frac{2.4}{7} \times 10{,}400$$

$$= 16{,}650''^{k} \quad \text{(POS. MOM. @ 11.7')}$$

$$M_u = \phi \left[A_s f_{su} d \; (1 - .59 q) \right]$$

OR $$M_u = \phi \left[.25 f'_c \, b d^2 \right]$$

DEPENDING ON WHETHER $P \dfrac{f_{su}}{f'_c}$ IS LESS THAN OR GREATER THAN .3

$$P = \frac{A_s}{bd} = \frac{4.18}{20d} \qquad d = e + y_t = 7.9 + 17.8 = 25.7$$

$$= .00812$$

$$f_{su} = f'_s \left(1 - .5 \frac{P f'_s}{f'_c}\right) = 250 \left(1 - .5 \; \frac{.00812 \times 250}{5}\right)$$

$$= 250 \quad (1 - .203) = 199 \text{ KIPS}$$

$$P \frac{f_{su}}{f'_c} = .00812 \; \frac{199}{5} = .323 \; > .3$$

BUT IF $f'_c = 6000$ LBS.

THEN $f_{su} = 250 \; (1 - .17) = 207$ KIPS

AND $P \dfrac{f_{su}}{f'_c} = .00812 \; \dfrac{207}{6} = .28 \; < .3$

FOR $f'_c = 6000$

$$M_u = .9 \left[4.18 \times 207 \times 25.7 \; (1 - .59 \times .28) \right]$$

$(\phi = .9$ FOR FLEXURE$)$

$$= 16{,}700''^{k} \quad > 16{,}650''^{k}$$

FOR $f'_c = 5000$

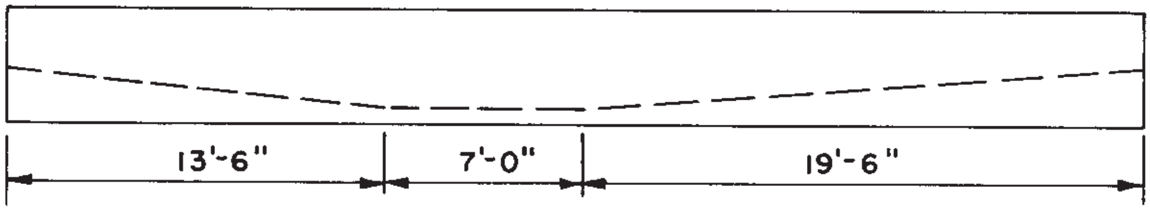
$$M_u = .9 \left[.25 \times 5 \times 20 \times (25.7)^2 \right]$$

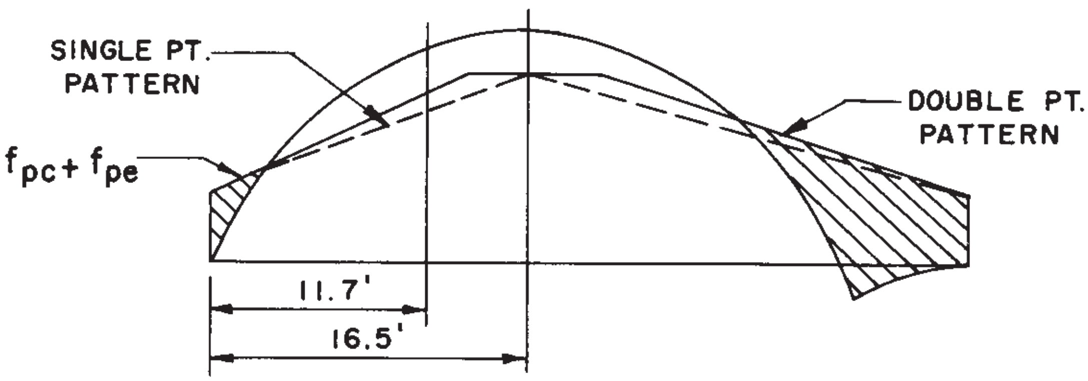
$$= 14{,}900''^{k} \quad < 16{,}650''^{k}$$

ONE POSSIBLE SOLUTION IS TO RAISE f'_c TO 6000 P.S.I.

ASSUME 5000 P.S.I. CONCRETE WILL BE USED.

TRY 2 POINT DEPRESSION OF STRANDS





BY INSPECTION, SECTION @ 11.7' NO LONGER CRITICAL

CHECK FINAL STRESSES AT MIDSPAN

$$f_{bf} = +\frac{F_f}{A} + \frac{F_{fe}}{S_b} - \frac{M_t}{S_b}$$ (f_{bf} = FINAL BOTTOM STRESS)

$$= +\frac{585}{.784} + \frac{585\,(11.2)}{4.8} - \frac{950\,(12)}{4.8}$$

$$= +745 + 1365 - 2375$$

$$= -265 < 425 \text{ P.S.I.}$$

$$f_{tf} = +\frac{F_f}{A} - \frac{F_{fe}}{S_t} + \frac{M_t}{S_t}$$ (f_{tf} = FINAL TOP STRESS)

$$= +\frac{585}{.784} - \frac{585\,(11.2)}{3.83} + \frac{950\,(12)}{3.83}$$

$$= +745 - 1710 + 2980$$

$$= +2015 \text{ P.S.I.} < 2250 = .45\,f'_c$$

INITIAL STRESSES AT 13.5' FROM LT. END (HOLD DOWN POINT)

$$f_{bi} = + \frac{Fi}{A} + \frac{Fie}{S_b} - \frac{M_D}{S_b} \qquad (f_{bi} = \text{INITIAL BOTTOM STRESS})$$

WHERE Fi = INITIAL PRESTRESSING FORCE AFTER ALLOWING FOR ELASTIC SHORTENING (ESTIMATED HERE AT 5%)

M_D = MOMENT AT SECTION DUE TO GIRDER WT. ALONE

$$f_{bi} = + \frac{.95 \times 730}{.784} + \frac{.95 \times 730 \times 11.2}{4.80} - \frac{\left(16.4^{K} \times 13.5' - \frac{.82 \times \overline{13.5}^2}{2}\right)12}{4.80}$$

$$= +885 + 1620 - 365$$

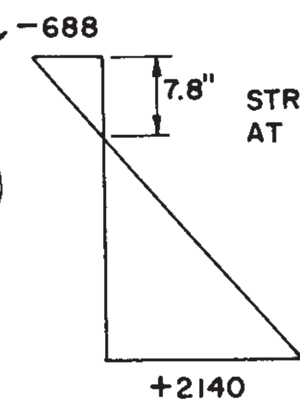
$$= +2140 \text{ P.S.I.} < 2400 = .60 f'_{ci}$$

$$f_{ti} = +885 - \frac{.95 \times 730 \times 11.2}{3.83} + \frac{1750}{3.83}$$

$$= +885 - 2030 + 457$$

$$= -688 > 3\sqrt{f'_{ci}} \text{ OR } 190 \text{ P.S.I.}$$

ACI 318-63 PLACES NO LIMIT ON THIS VALUE, BUT IS ACCEPTED HERE FOR ILLUSTRATIVE PUPPOSES. THE AUTHOR RECOMMENDS FOR MOST CASES THAT THE TENSILE STRESS NOT EXCEED 500 P.S.I.



STRESS DIAGRAM AT 13.5' FROM LT.

$$\text{TENSILE FORCE} = 688 \times 20 \times 7.8 \times \frac{1}{2} = 53.6 \text{ KIPS}$$

$$A'_s = \frac{53.6}{20} = 2.68 \text{ in.}^2 \qquad (\text{USE } 3 \text{ \# } 9 \text{ BARS})$$

INITIAL STRESSES AT ENDS

O.K. BY INSPECTION SINCE e = 0 AND M_D = 0, LAST TWO TERMS IN ABOVE EQUATIONS DROP OUT.

ULTIMATE MOMENT AT MIDSPAN

$$Mu = 1.5 \times \frac{4.6}{7} \times 950 + 1.8 \times \frac{2.4}{7} \times 950$$

$$= 936 + 586$$

$$= 1522'^{k} = 18{,}250''^{k}$$

MOMENT CAPACITY AT MIDSPAN

$$P = \frac{4.18}{20(29)} = .00722$$

$$f_{su} = 250 \left(1 - .5 \frac{.00722 \times 250}{5}\right) = 205^{ksi}$$

$$q = \frac{205\,(.00722)}{5}$$

$$= .296 < .3$$

USE

$$Mu = \phi \left[As\, f_{su} d\,(1 - .59q)\right] \quad \text{WITH } \phi = .9$$

$$Mu = .9 \left[4.18 \times 205 \times 29 \;(1 - .59 \times .296)\right]$$

$$= 18{,}400''^{k} > 18{,}250''^{k}$$

CHECK STRESSES AT CANTILEVER SUPPORT

$$e = \frac{6}{19.5}(11.2) = 3.45$$

$$d = 14.2 - 3.5 = 10.7 \text{ in.}$$

$$f_{bf} = + \frac{585}{.784} + \frac{585\,(3.5)}{4.8} + \frac{126\,(12)}{4.8}$$

$$= +745 + 427 + 315$$

$$= +1487 \text{ P.S.I.} < 2250$$

$$f_{tf} = + \frac{585}{.784} - \frac{585\,(3.5)}{3.83} - \frac{126\,(12)}{3.83}$$

$$= -186 \text{ P.S.I.} < 425$$

CHECK ULTIMATE MOMENT AT CANTILEVER SUPPORT

$$M_u = 1.5\ \frac{4.6}{7}(126) + 1.8\ \frac{2.4}{7}(126)$$

$$= 124 + 78$$

$$= 202'^{k} = 2425''^{k}$$

MOMENT CAPACITY AT CANTILEVER

$$P = \frac{4.18}{32(10.7)} = .0122$$

$$f_{su} = 250\left[1 - .5\ \frac{.0122 \times 250}{5}\right] = 174 \text{ ksi}$$

$$q = \frac{174}{5}(.0122)$$

$$= .43 > .3$$

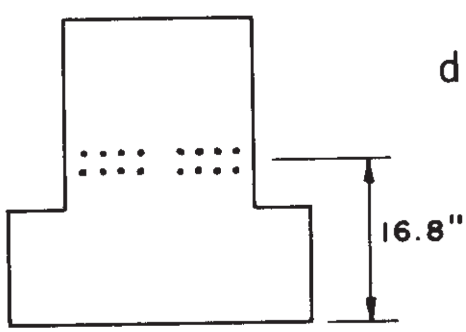
USE

$$M_u = \phi\left[.25 f'_c\, bd^2\right] \quad \text{WITH} \quad \phi = .9$$

$$= .9 \times .25 \times 5 \times 32 \times \overline{10.7}^2$$

$$= 4120''^{k} > 2425''^{k}$$

ASSUME TOP ROW OF STRANDS ONLY ARE EFFECTIVE



$$d \text{ @ SUPPORT} = 3'' + \frac{13.5}{19.5}(23-3) = 16.8''$$

$$A_s = 16(.144) = 2.30 \text{ in}^2$$

$$P = \frac{2.30}{32(16.8)} = .00428$$

$$f_{su} = 250\left(1 - .5\ \frac{.00428 \times 250}{5}\right) = 223 \text{ ksi}$$

$$q = \frac{221}{5}(.00428) = .19 < .3$$

$$M_u = .9\ (2.30 \times 223 \times 16.8\ (1 - .59 \times .19)$$

$$= 6880''^{k} > 2425''^{k}$$ AND OVER 60% $> 4120''^{k}$ COMPUTED USING ALL STRANDS.

CHECK BOND AT CANTILEVER SUPPORT

f_{su} = 223 ksi

f_{se} = 140 ksi (SEE P.71)

REQUIRED BOND LENGTH $= (f_{su} - \frac{2}{3} f_{se})D$

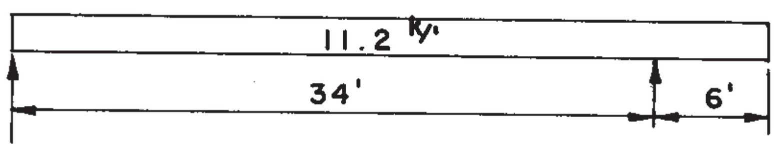
$= (223 - \frac{2}{3}(140).5$

$= 65'' < 72''$ TO SUPPORT

WHERE D IS DIAMETER OF STRAND

SHEAR

COMPUTE ULTIMATE SHEAR



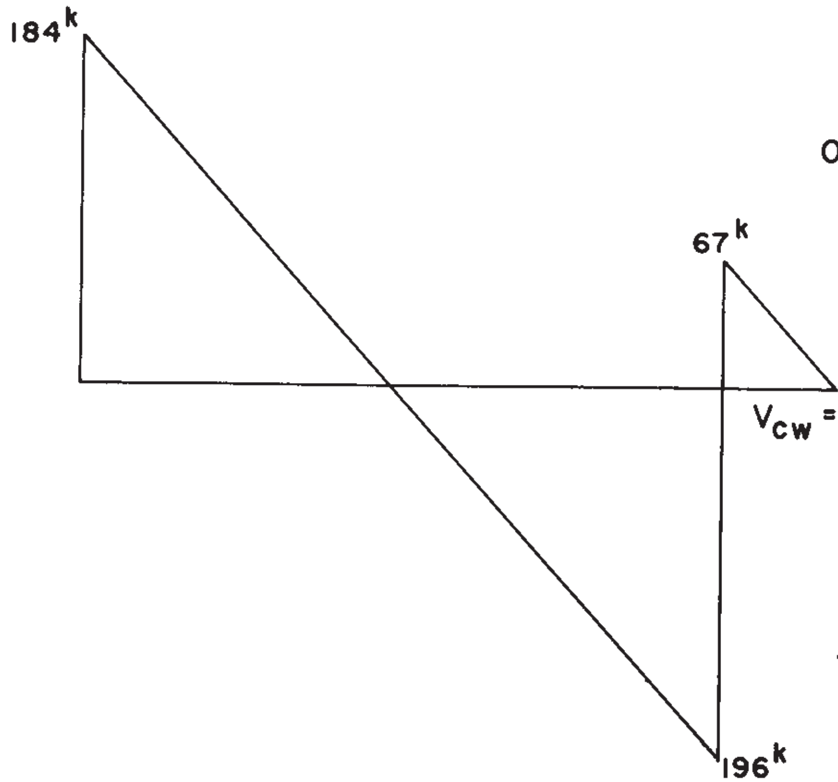
$W_D = 4.6^{K/'}$

$W_L = 2.4^{K/'}$

$W_u = 1.5\ (4.6)$

$\quad + 1.8\ (2.4)$

$\quad = 11.2^{K/'}$



$b' = 20''$

$d = 29''$ @ ℄

OR $d = .8(32) = 25.6''$

$f'_c = 5000$ P.S.I.

$F_f = 585$ KIPS

$V_{cw} = b'd(3.5\sqrt{f'_c} + .3\frac{F_f}{A}) + V_p$

$= 20 \times 25.6 \times 3.5\ \sqrt{5000}$

$+ 20 \times 25.6 \times .3\ \frac{585}{.784}$

$+ \frac{11.2''}{162''}(585)$

$= 281^k$

$$V_{ci} = .6\, b'd \sqrt{f'_c} + \frac{M_{cr}}{\frac{M}{V} - \frac{d}{2}} + V_d$$

BUT NOT LESS THAN $1.7\, b'd \sqrt{f'_c}$

WHERE

$$M_{cr} = \frac{I}{y}(6\sqrt{f'_c} + f_{pe} - f_d)$$

$$= S_b\left(6\sqrt{f'_c} + \frac{F_f}{A} + \frac{F_{fe}}{S_b} - \frac{M_D}{S_b}\right)$$

$$V_{cw} = b'd\,(3.5\sqrt{f'_c} + .3\, f_{pe}) + V_p$$

AT SECTION 5' FROM LEFT END

$$V_u = 184 - 11.2\,(5) = 128^k$$

$$M_u = 184\,(5) - \frac{11.2(5)^2}{2} = 780'^k = 9400''^k$$

$$V_D = \frac{4.6}{11.2}(128) = 52.8^k$$

$$\frac{M}{V} = \frac{9400}{128} = 73''$$

$$e = \frac{5}{13.5}(11.2'') = 4.15''$$

$$M_D = \frac{4.6}{11.2}(9400) = 3850''^k$$

$$M_{cr} = 4.8\left(425 + \frac{585}{.784} + \frac{585(4.15)}{4.8} - \frac{3850}{4.8}\right)$$

$$= 2040 + 3580 + 2430 - 3850$$

$$= 4200''^k$$

$$V_{ci} = .6 \times 20 \times 22\sqrt{5000} + \frac{4200}{73 - \frac{22}{2}} + 52.8$$

$$= 19 + 68 + 53$$

$$= 140^k$$

AT SECTION 10' FROM LEFT END

$$V_u = 184 - 11.2\,(10) = 72^k$$

$$M_u = 184\,(10) - 11.2\,\frac{10^2}{2} = 1280'^k = 15{,}400''^k$$

$$V_D = \frac{4.6}{11.2}(72) = 29.5^k$$

$$\frac{M}{V} = \frac{15,400}{72} = 214''$$

$$e = \frac{10}{13.5}(11.2'') = 8.3''$$

$$M_D = 15,400\ \frac{4.6}{11.2} = 6300''^k$$

$$M_{cr} = 4.8\left(425 + \frac{585}{.784} + \frac{585\,(8.3)}{4.8} - \frac{6300}{4.8}\right)$$

$$= 2040 + 3580 + 4850 - 6300$$

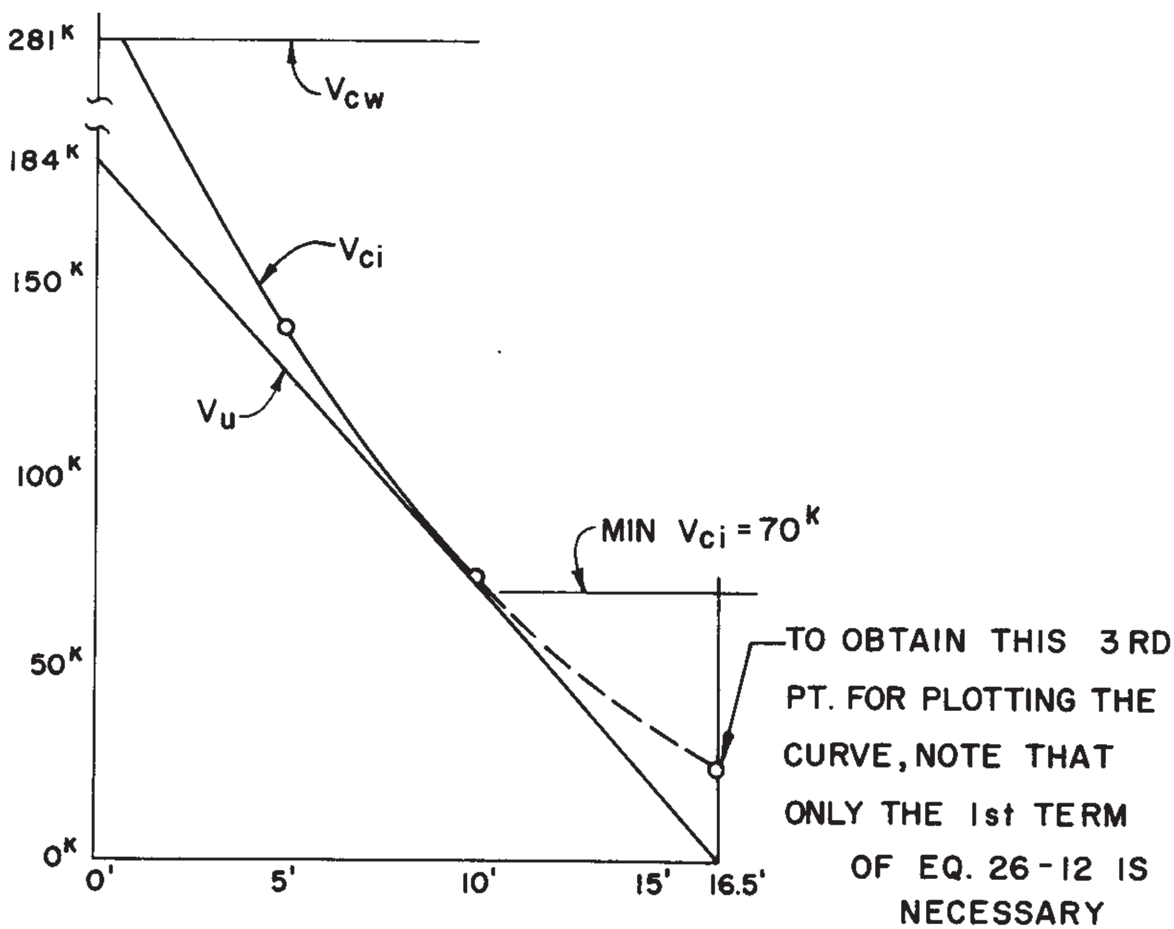
$$= 4170''^k$$

$$V_{ci} = .6 \times 20 \times 26 \times \sqrt{5000} + \frac{4170}{214 - \frac{26}{2}} + 29.5$$

$$= 22 + 21 + 29.5$$

$$= 73^k$$

$$\text{OR } V_{ci} = 1.7(20)(29)\sqrt{5000} = 70^k$$



$$A_v = \frac{(V_u - \phi V_c)\,s}{\phi\, d\, f_y} \qquad \text{WHERE } \phi = .85 \text{ FOR SHEAR}$$

$$A_v \text{ @ } 10' = \frac{(72 - .85 \times 73)\,S}{.85 \times 29 \times 40} = .0102\ S$$

MINIMUM REQUIREMENT

$$A_v = \frac{A_s}{80} \cdot \frac{f_s'}{f_y} \cdot \frac{S}{d}\sqrt{\frac{d}{b'}}$$

$$= \frac{4.18}{80} \cdot \frac{250}{40} \cdot \frac{S}{29}\sqrt{\frac{29}{20}}$$

$$= .0135\ s \quad \therefore \text{ MIN. REQ. GOVERNS}$$

$$S \leqq \frac{3}{4} d \text{ OR } \leqq 24''$$

$$\frac{3}{4} d = .75\,(29) = 22''$$

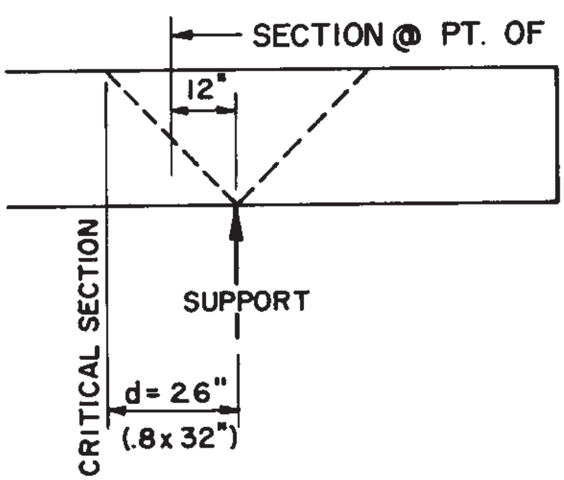
USE # 3 ⊔ STIRRUPS @ 16" FULL LENGTH

WITH 3 @ 8" EACH END

A_v REG. @ 16" = .0135 (16) = .22 in.^{2}

A_v FURNISHED = 2 (.11) = .22 in^2

CHECK REGION OF CANTILEVER SUPPORT



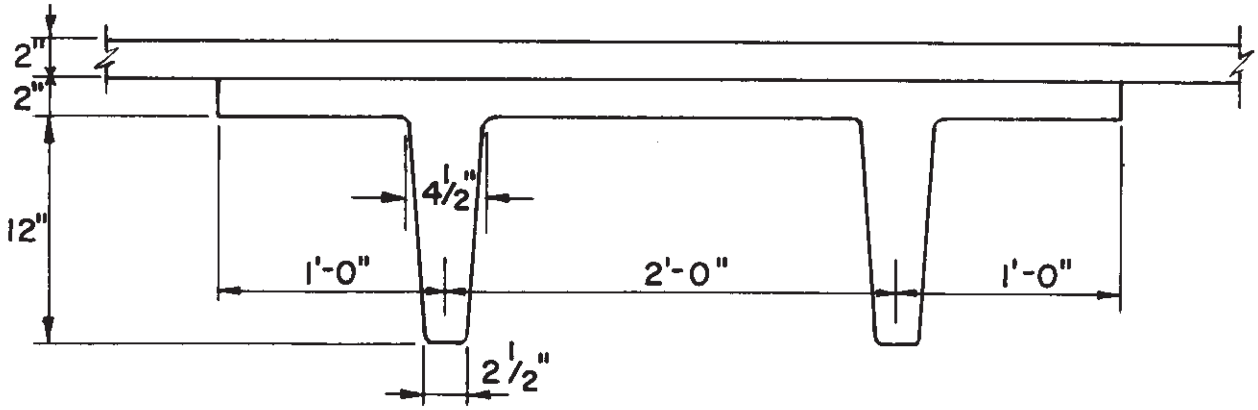
CRITICAL SECTION $- \frac{d}{2}$

= PT. OF CONTRAFLEXURE (APPROX.)

$\therefore\ \frac{M}{V} - \frac{d}{2}$ = APPROX. 0 , SINCE M = 0

THEN V_{ci} = APPROX. 00

COMPOSITE SECTION – DOUBLE TEE AND SLAB



PROPERTIES	AREA	WEIGHT	I	Y_b	f'_c
PRECAST	180	47	2864	10.00	5000
COMPOSITE	96	25	4460 *	11.75 *	3000
	SQ. IN.	P.S.F.	$IN.^4$	IN.	P.S.I.

* IF CORRECTED FOR CONCRETE STRENGTHS, I = 4203 . Y_b = 11.45"

DESIGN THE COMPOSITE SECTION ABOVE FOR A CEILING LOAD OF 10 P.S.F. AND A LIVE LOAD OF 60 P.S.F. ON A SIMPLE SPAN OF 30'-0". USE 3/8" STRAND WITH f'_s = 250,000 P.S.I.

MAX. SHEAR & MOM.	SHEAR - PRECAST	- MOM.	SHEAR - COMP.	- MOM.
DEAD LOAD	2820	254,000		
TOPPING SLAB	1500	135,000		
CEILING			600	54,000
LIVE LOAD			3600	324,000
	#	"#	#	"#

DEAD LOAD: $f_b = \dfrac{254000\,(10.00)}{2864} = 886$ P.S.I.

$f_t = \dfrac{254000\,(4.00)}{2864} = 354$ P.S.I.

TOPPING SLAB: $f_b = \dfrac{135000\,(10.00)}{2864} = 471$ P.S.I.

$f_t = \dfrac{135000\,(4.00)}{2864} = 188$ P.S.I.

CEILING: $f_b = \dfrac{54000\,(11.75)}{4460} = 142$ P.S.I.

$f_t = \dfrac{54000\,(2.25)}{4460} = 27$ P.S.I.

$f_{tc} = \dfrac{54000\,(4.25)}{4460} = 51$ P.S.I.

LIVE LOAD:

$$f_b = \frac{324000\ (11.75)}{4460} = 853 \text{ P.S.I.}$$

$$f_t = \frac{324000\ (2.25)}{4460} = 164 \text{ P.S.I.}$$

$$f_{tc}\ \frac{324000\ (4.25}{4460} = 309 \text{ P.S.I.}$$

$f_b = 886 + 471 + 142 + 853 = 2352$ P.S.I.

$f_t = 354 + 188 + 27 + 164 = 733$ P.S.I.

MIN. $f_b = 2352 - 6\sqrt{f'c} = 2352 - 424 = 1928$ P.S.I.

MAX. $f_t = 0.45\ f'c - 733 = 2250 - 733 = 1517$ P.S.I.

USE 3 - 3/8" STRANDS PER STEM WITH CENTER POINT DEPRESSION AND 1 1/2" CLEAR.

$e = 10.00 - (1.50 + 0.375 + 0.1875) = 7.94"$

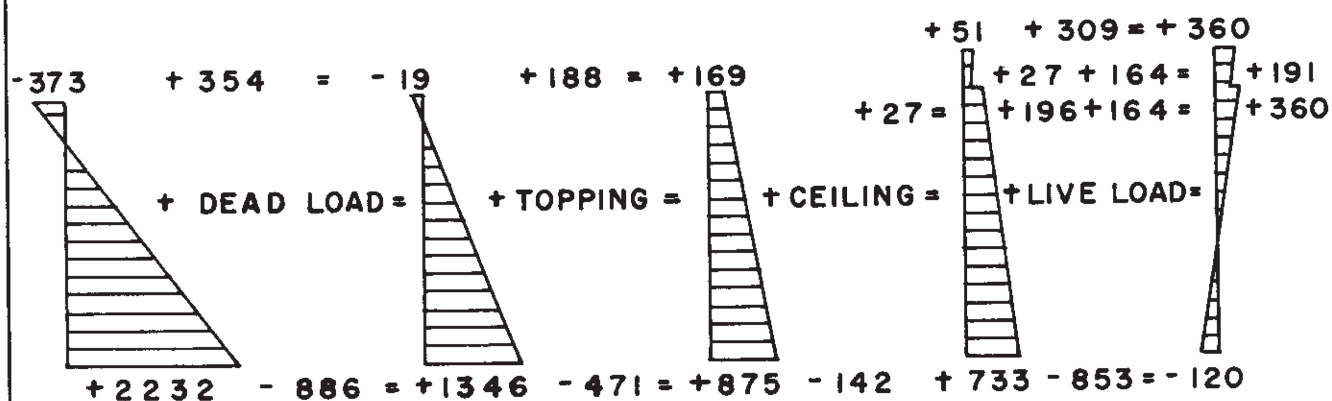
AT MID-SPAN AFTER LOSSES: (ASSUME 20% LOSS)

$$f_b = \frac{6\ (11200)}{180} + \frac{6\ (11200)\ (7.94)\ (10.00)}{2864} =$$

$372 + 1860 = +\ 2232$ P.S.I. (COMPRESSION)

$$f_t = \frac{6\ (11200)}{180} - \frac{6\ (11200)\ (7.94)\ (4.00)}{2864} =$$

$372 - 745 = -\ 373$ P.S.I. (TENSION)



AT TRANSFER AT MID-SPAN:

$f_{ti} = [-373\ (1.25) = -466] + 354 = -112$ P.S.I.

$f_{bi} = [+2232\ (1.25) = +2790] - 886 = +1904$

STRENGTH REQUIRED AT TRANSFER:

$f_{bi} = 0.60\ f'_{ci}$ $\qquad f'_{ci} = \frac{1904}{0.60} = 3180$ P.S.I.

$f_{ti} = 3\sqrt{f'_{ci}}$ $\qquad f'_{ci} = \left(\frac{112}{3}\right)^2 = 1400$ P.S.I.

MINIMUM RELEASE STRENGTH = 3200 P.S.I.

AT ENDS, USE STRAND SPACING OF 3" - 6" - 9"

$e = 10.00 - 6.00 = 4.00"$

$f_b = 372 + \frac{4.00}{7.94}(1860) = 372 + 937 = 1309$ P.S.I.

AT TRANSFER $f_b = 1.25(1309) = +1635$ P.S.I.

$f_t = 372 - \frac{4.00}{7.94}(745) = 372 - 375 = -3$ P.S.I.

AT TRANSFER = -4 P.S.I.

ULTIMATE STRENGTH:

$p = \frac{A_s}{bd} = \frac{6\ (0.08)}{48\ (13.94)} = \frac{0.48}{670} = 0.00072$

$f_{su} = f'_s\ (1 - 0.5\ \frac{pf'_s}{f'_c}) = 250{,}000\ \left[1 - 0.5\ \frac{0.00072\ (250)}{3}\right]$

$= 242{,}500$ P.S.I.

$q = \frac{pf_{su}}{f'_c} = \frac{0.00072\ (242.5)}{3} = 0.058$

$1.4\,dq = 1.13"$ ($< 4" \therefore$ FORMULAS FOR RECT. SEC. APPLY)

$M_u = \phi\ [A_s f_{su} d\ (1 - 0.59\ q)\ ]$

$= 0.90\ (0.48)\ (242{,}500)\ (13.94)\ [1 - 0.59\ \ (0.058\ ]$

$= 1{,}410{,}000"\# \times 0.95^{*} = 1{,}340{,}000"\#$

$U = 1.5\,D + 1.8\,L = 1.5\ (443{,}000) + 1.8\ (324{,}000)$

$= 1{,}247{,}700"\#$

BOND DISTANCE: $f_{se} = 0.7\ f'_s - 35 = 0.7\ (250) - 35 = 140$ ksi

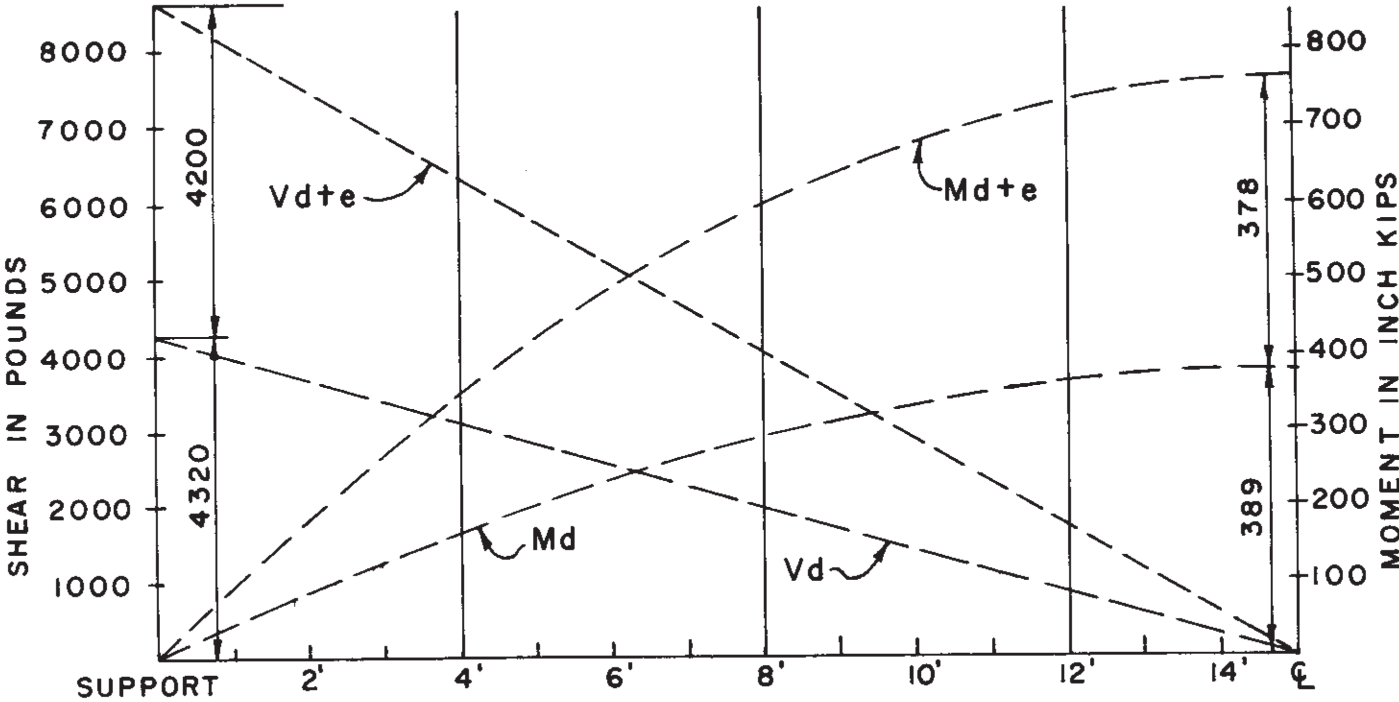
DIST. $= D\ (f_{su} - \frac{2}{3}\ f_{se}) = \frac{3}{8}\ \left[242.5 - \frac{2\ (140)}{3}\right] = 56" < 192"$

1/2 (SPAN + BEARING)

ULTIMATE SHEAR: (NOTE: THESE CALCULATIONS ARE FOR A LONG SHEAR INVESTIGATION - THE SHORTER INVESTIGATION OUTLINED IN THE SECTION ON SHEAR ARE RECOMMENDED.)

$V_u = 1.5D + 1.8L = 1.5\ (4920) + 1.8\ (3600) = 13{,}860\#$

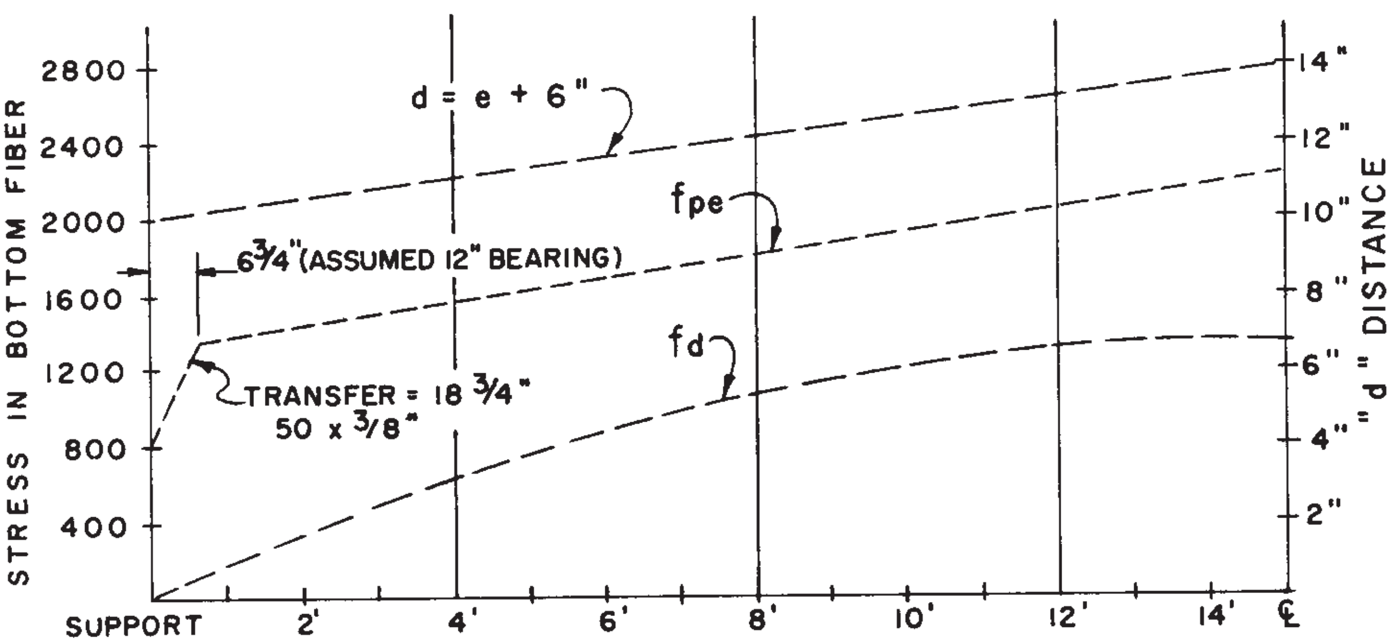
$*$ REDUCTION FACTOR FOR MID-PT. DEPRESSION



POINT	M *	V*	M / V	Vd +	d	d/2
4'	175	3.08	56.8	3.17	11.05	5.52
8'	296	1.96	151.0	2.02	12.10	6.05
12'	363	0.84	432.0	0.86	13.15	6.58

*DUE TO EXTERNAL LOADS - CEILING + L L

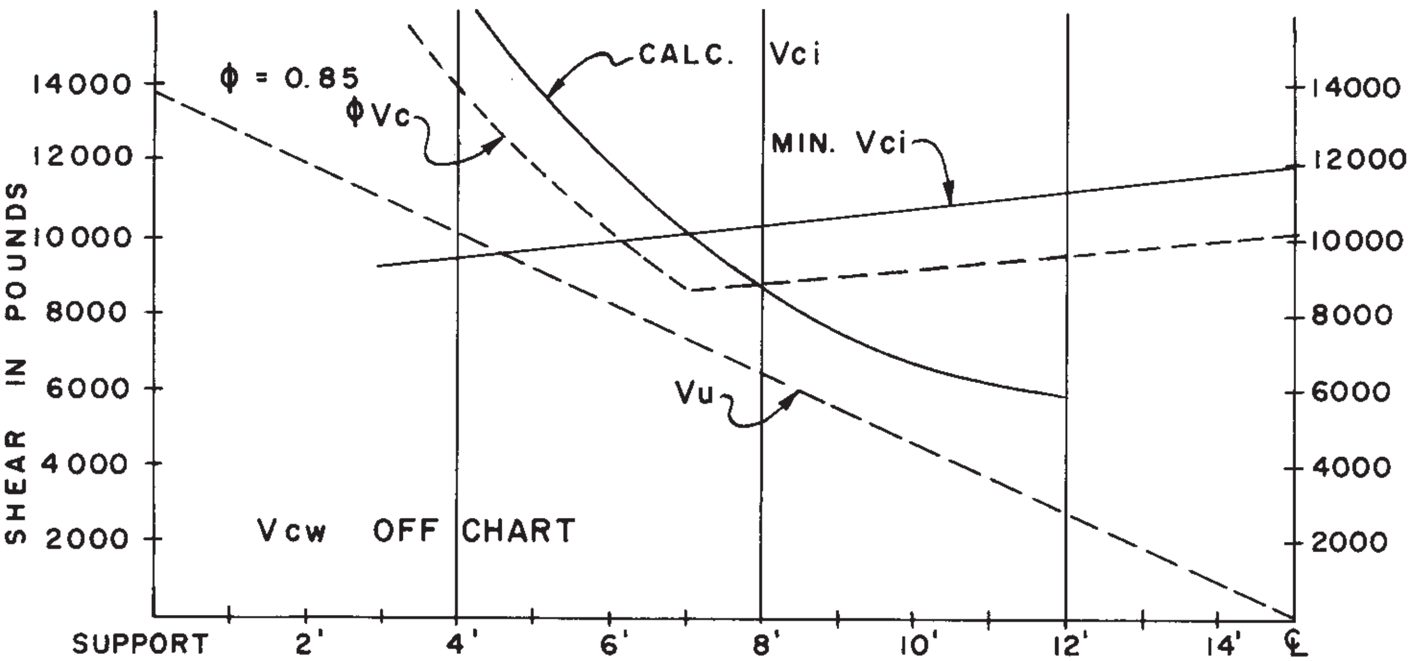
+ DUE TO BEAM + SLAB



@ 4', fpe - fd = 1550 - 625 = 925

@ 8', 1800 - 1065 = 735

@ 12, 2050 - 1300 = 750



$$\text{MIN. } A_v = \frac{A_s}{80} \cdot \frac{f'_s}{f_y} \cdot \frac{s}{d} \sqrt{\frac{d}{b'}}$$

$$= \frac{0.48}{80} \cdot \frac{250}{40} \cdot \frac{12}{13.94} \sqrt{\frac{13.94}{7.0}}$$

$$= 0.046 \qquad \text{TOTAL} = 0.023 \ \square''/\text{STEM}$$

#2 STIRRUPS @ 12" = 0.05 □"/STEM

6 x 6 - 10 / 10 MESH = 0.029 □"/STEM

CHECK v_h AT CONTACT PLANE : (NO SHORES USED)

$Q = 2\ (48)\ (3.25) = 312$ $\qquad V = V_u - V_d = 13860 - 4320 = 9540$

$$v_h = \frac{VQ}{Ib} = \frac{9540\ (312)}{4460\ (48)} = 14 \text{ P.S.I. @ ULTIMATE}$$

ALLOWABLE $v_h = 1.9\ (40) = 76$ P.S.I.

$$M_{CR} = \frac{I}{y}\left(6\sqrt{f'_c} + f_{pe} - f_d\right) = \frac{4460}{11.75}\left[6\sqrt{5000} + (f_{pe} - f_d)\right]$$

$$= 380\ \left[424 + (f_{pe} - f_d)\right]$$

POINT	$f_{pe} - f_d$	+ 424	x 380 = M_{CR}	$\frac{M}{V} - \frac{d}{2}$	Vd
4'	925	1349	513000	51.3	3168
8'	735	1159	441000	145.0	2016
12'	750	1174	446000	425.5	864

$$V_{ci} = 0.6\ b'd\ \sqrt{f'_c} + \frac{M_{CR}}{\frac{M}{V} - \frac{d}{2}} + V_d \qquad V_{ci} = 1.7\ b'd\ \sqrt{f'_c}$$

$$V_{ci} @ 4' = 0.6\ (7.0)(11.05)\ \sqrt{5000} + \frac{513000}{51.3} + 3168$$
$$= 16{,}450^{\#}$$

$$\text{MIN. } V_{ci} @ 4' = 1.7\ (7.0)(11.05)\ \sqrt{5000} = 9{,}300^{\#}$$

$$V_{ci} @ 8' = 0.6\ (7.0)(12.10)\ \sqrt{5000} + \frac{441000}{145} + 2016$$
$$= 8{,}650^{\#}$$

$$\text{MIN. } V_{ci} @ 8' = 1.7\ (7.0)(12.10)\ \sqrt{5000} = 10{,}200^{\#}$$

$$V_{ci} @ 12' = 0.6\ (7.0)(13.15)\ \sqrt{5000} + \frac{446000}{425.5} + 864$$
$$= 5810^{\#}$$

$$\text{MIN. } V_{ci} @ 12' = 1.7\ (7.0)(13.15)\ \sqrt{5000} = 11{,}080^{\#}$$

$$\text{MIN. } V_{ci} @ 15' = 1.7\ (7.0)(13.94)\ \sqrt{5000} = 11{,}750^{\#}$$

$$f_{pc} @ \mathrm{C\!\!\!L} = 169 + \frac{2.25}{14}(875 - 169) = 282 \text{ P.S.I.}$$

$$@ \text{ SUPPORT} = -3 + \frac{2.25}{14}(1309 + 3) = 208 \text{ P.S.I.}$$

$$V_p = \frac{3.94}{15(12)}(6)(11200) = 1470^{\#}$$

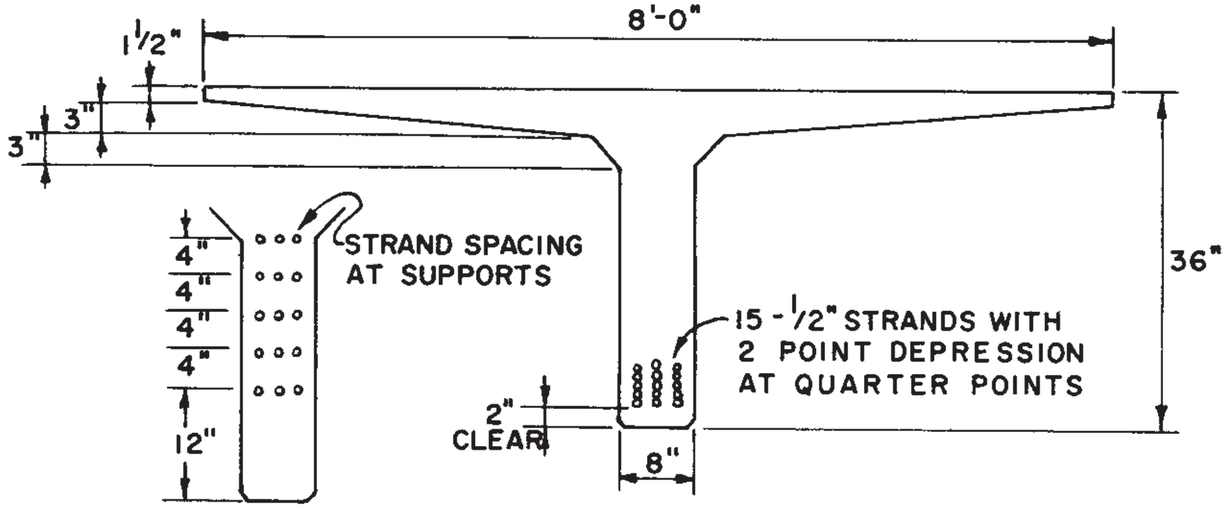
d = 0.8 (16) = 12.8" (MIN. "d" USED)

$$V_{cw} = b'd\ (3.5\ \sqrt{f'_c} + 0.3\ f_{pc}) + V$$

$$= (7.0)(12.8)\left[3.5\ (70.7) + 0.3\ (208)\right] + 1470$$

$$= 29{,}250^{\#} @ \text{ SUPPORT}$$

TEE BEAM



AREA	WT (psf)	I (in^4)	Yb (in.)	f'_c (psi)	f'_s (psi)
561	73	68750	25.9	5000	250,000

THIS SECTION WAS DESIGNED FOR A ROOFING LOAD OF 7 P.S.F. (NO CEILING) AND A LIVE LOAD OF 30 P.S.F. ON A SIMPLE SPAN OF 80'-0". THE TENSIONING LOAD PER STRAND IS 25,200 #. AFTER 20% LOSS, DESIGN LOAD IS 20,160 #. AREA IS 0.144 SQ.IN. PER STRAND. CLOSED # 3 STIRRUPS ARE SPACED AT 20" FOR THE FULL LENGTH OF THE MEMBER.

INVESTIGATE:
1. STRESSES UNDER FULL LOAD AT MID-SPAN AFTER LOSSES.
2. STRESSES AT TRANSFER AT QUARTER POINT.
3. ULTIMATE FLEXURAL CAPACITY AT MID-SPAN.
4. SHEAR CAPACITY AND STIRRUPS 10' FROM SUPPORT.

MAX. SHEAR & MOM.	SHEAR (#)	MOMENT ("#)
DEAD LOAD	23,400	5,610,000
ROOFING	2,200	538,000
LIVE LOAD	9,600	2,305,000

DEAD LOAD: $f_b = \dfrac{5610000\ (25.9)}{68750} = 2115$ P.S.I.

$f_t = \dfrac{5610000\ (10.1)}{68750} = 825$ P.S.I.

ROOFING: $f_b = \dfrac{538000\ (25.9)}{68750} = 203$ P.S.I.

$f_t = \dfrac{538000\ (10.1)}{68750} = 79$ P.S.I.

LIVE LOAD: $f_b = \dfrac{2305000\ (25.9)}{68750} = 869$ P.S.I.

$f_t = \dfrac{2305000\ (10.1)}{68750} = 339$ P.S.I.

$e = 25.9 - (2.00 + 0.50 + 0.50 + 0.25) = 22.65''$

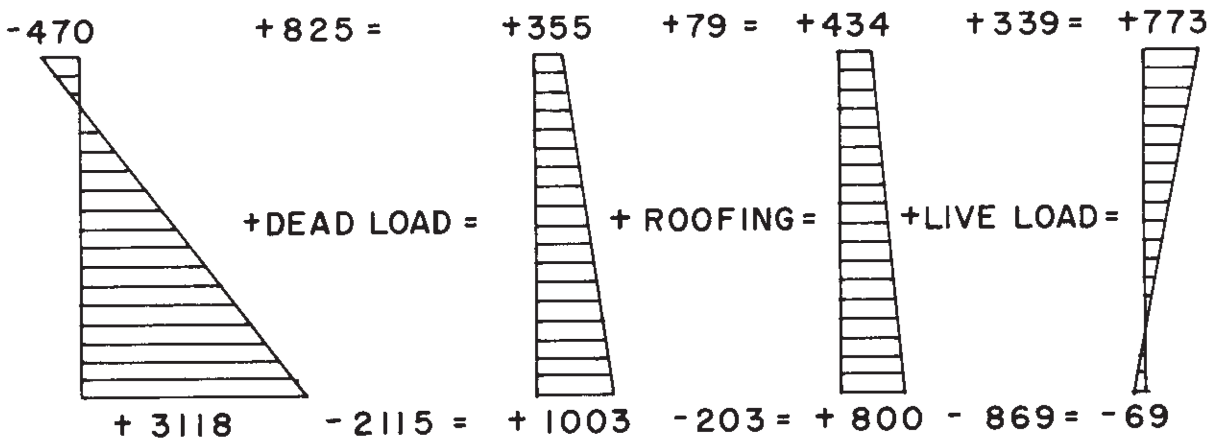
AT MID-SPAN AFTER 20% LOSS.

$$f_b = \frac{15\ (20160)}{561} + \frac{15\ (20160)(22.65)(25.9)}{68750} =$$

$538 + 2580 = +3118$ P.S.I.

$$f_t = \frac{15\ (20160)}{561} - \frac{15\ (20160)(22.65)(10.1)}{68750} =$$

$538 - 1008 = -470$ P.S.I.



AT QUARTER-POINT AT TRANSFER:

$f_{ti} = -470\ (1.25) = -588 + \frac{3}{4}\ (825) = +31$ P.S.I.

$f_{bi} = +3118\ (1.25) = +3900 - \frac{3}{4}\ (2115) = +2320$ P.S.I.

MIN. RELEASE: $f'_{ci} = \dfrac{2320}{0.60} = 3870$, SAY 3900 P.S.I.

AT SUPPORTS, $e = 25.9 - 20.0 = 5.9''$

$$f_b = 538 + \frac{5.9}{22.65}(2580) = 538 + 671 = +1209 \text{ P.S.I.}$$

@ TRANSFER, $f_b = 1.25(1209) = +1510$ P.S.I.

$$f_t = 538 - \frac{5.9}{22.65}(1008) = 538 - 262 = +276 \text{ P.S.I.}$$

@ TRANSFER = + 345 P.S.I.

ULTIMATE STRENGTH:

$$p = \frac{A_s}{bd} = \frac{15\ (0.144)}{96\ (32.75)} = \frac{2.16}{3140} = 0.00069$$

$$f_{su} = f'_s\left(1 - 0.5\frac{p\,f'_s}{f'_c}\right) = 250{,}000\left[1 - 0.5\ \frac{0.00069\ (250)}{5}\right]$$

$= 245{,}000$ P.S.I.

$$q = \frac{p\,f_{su}}{f'_c} = \frac{0.00069\ (245)}{5} = 0.0338$$

$1.4\,dq = 1.56''$
(CLOSE TO $1.50''$ SO USE EQ. FOR RECT. SECT.)

$$M_u = \phi\left[A_s f_{su}\ d\ (1 - 0.59\,q)\right]$$

$$= 0.90\left[(2.16)\,(245{,}000)\,(32.75)\ \ (1 - 0.59 \times 0.0338)\right]$$

$= 15{,}200{,}000''\#$

$$U = 1.5\,D + 1.8\,L = 1.5\ (6{,}148{,}000) + 1.8\ (2{,}305{,}000)$$

$= 13{,}371{,}000''\#$

BOND DISTANCE:

$$\text{DIST.} = D\left(f_{su} - \tfrac{2}{3} f_{se}\right) = \frac{1}{2}\left[245 - \frac{2\ (140)}{3}\right] = 76'' < 492''$$

$\frac{1}{2}$ (SPAN + BEARING)

ULTIMATE SHEAR:

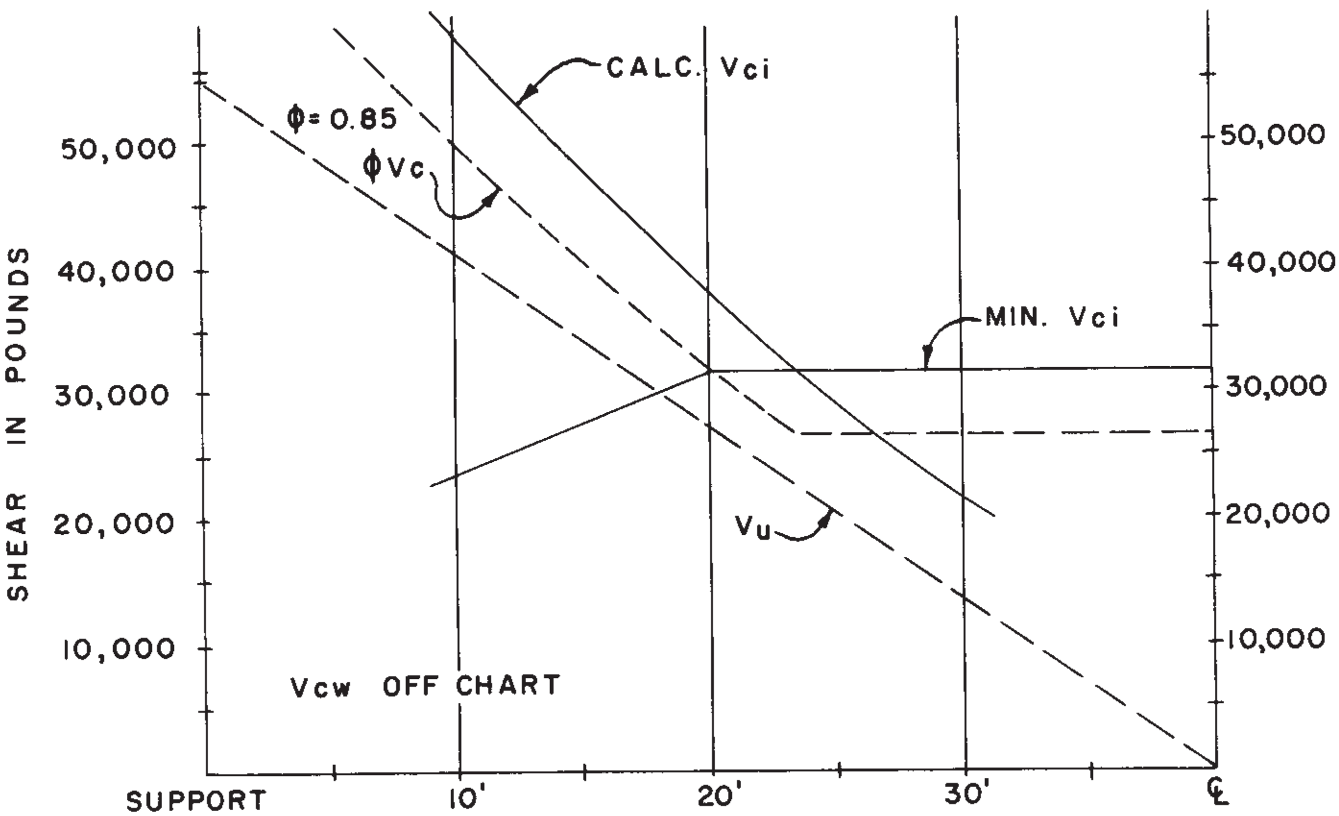
$$V_u = 1.5\,D + 1.8\,L = 1.5\ (25{,}600) + 1.8\ (9600)$$

$= 55{,}680\#$

$$M_{CR} = \frac{I}{y}\left(6\sqrt{f'_c} + f_{pe} - f_d\right)$$

$$= \frac{68750}{25.9}\left[6\sqrt{5000} + (f_{pe} - f_d)\right]$$

$$= 2665\ \left[424 + (f_{pe} - f_d)\right]$$

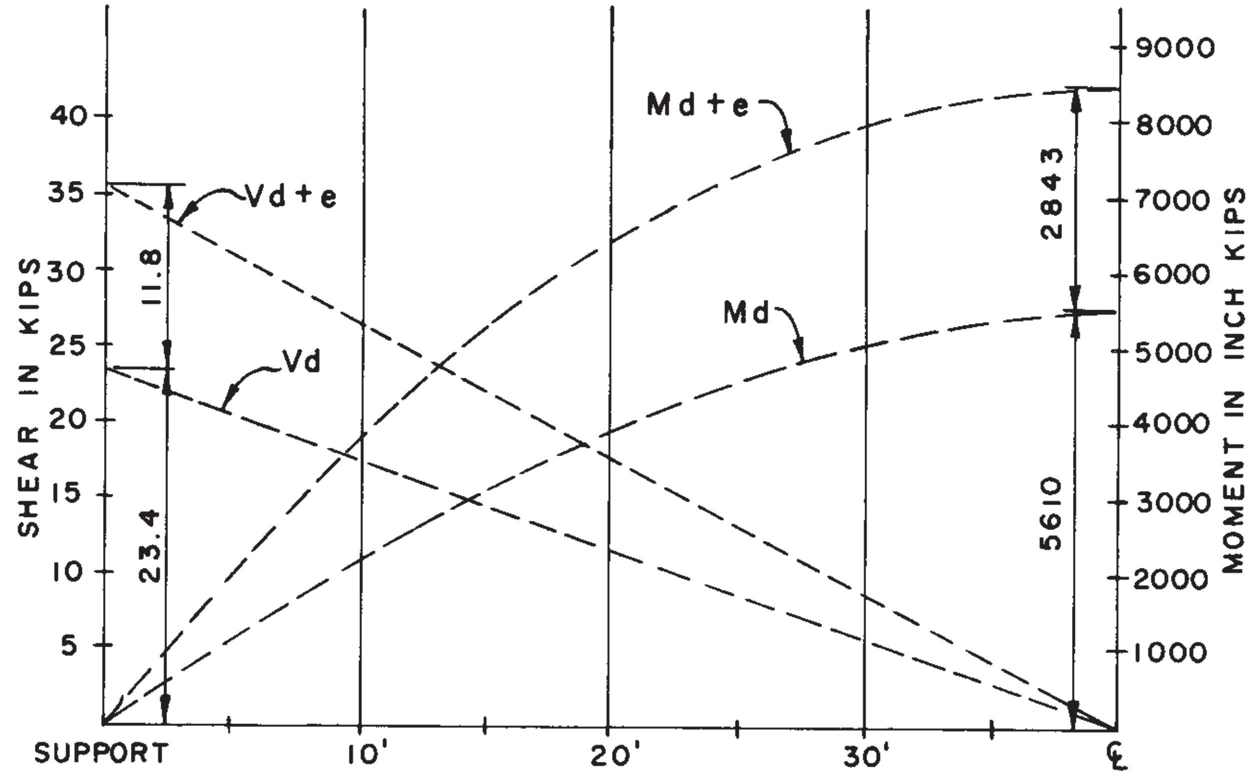


$$\text{MIN. } A_v = \frac{A_s}{80} \cdot \frac{f'_s}{f'_y} \cdot \frac{s}{d} \sqrt{\frac{d}{b'}}$$

$$= \frac{2.16}{80} \cdot \frac{250}{40} \cdot \frac{20}{32.75} \sqrt{\frac{32.75}{8}}$$

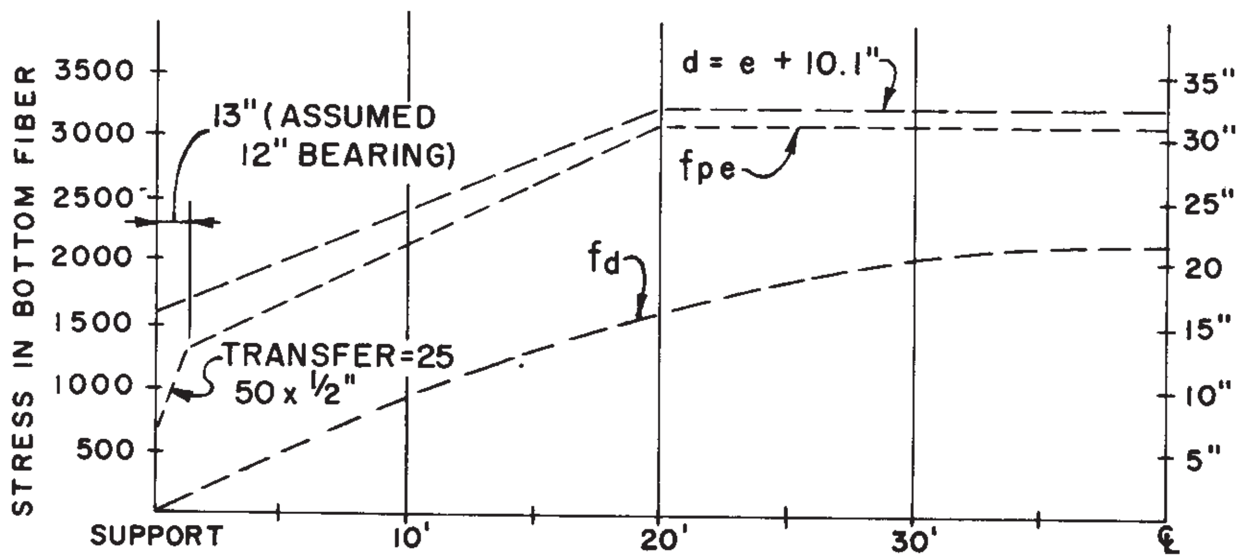
$$= 0.209 \, \square''$$

\# 3 ▯ STIRRUPS @ 20" = 0.22 □"



POINT	M*	V*	M/V	Vd†	d	d/2
10'	1250	8.85	141	17.55	24.37	12.18
20'	2133	5.90	360	11.70	32.75	16.37
30'	2660	2.95	903	5.85	32.75	

*DUE TO EXTERNAL LOADS - ROOFING + LL   †DUE TO BEAM ONLY



@ 10', fpe - fd = 2163 - 922 = 1241
@ 20', 3118 - 1585 = 1533
@ 30', 3118 - 1980 = 1138

POINT	fpe - fd	+ 424	x 2655 = M_{CR}	$\frac{M}{V} - \frac{d}{2}$	Vd
10'	1241	1665	4,420,000	129	17550
20'	1533	1957	5,200,000	344	11700
30'	1138	1562	4,150,000	887	5850

(SEE SHEAR SECTION FOR DISCUSSION OF SHORTER INVESTIGATION)

$$V_{ci} = 0.6\ b'd\ \sqrt{f'_c} + \frac{M_{CR}}{\frac{M}{V} - \frac{d}{2}} + V_d$$

$$V_{ci} = 1.7\ b'd\ \sqrt{f'_c}$$

$$V_{ci} \ @\ 10' = 0.6\ (8)\ (24.37)\ \sqrt{5000} + \frac{4420000}{129} + 17550$$
$$= 60{,}100^{\#}$$

$$\text{MIN. } V_{ci} \ @\ 10' = 1.7\ (8)\ (24.37)\ \sqrt{5000} = 23{,}400^{\#}$$

$$V_{ci} \ @\ 20' = 0.6\ (8)\ (32.75)\ \sqrt{5000} + \frac{5200000}{344} + 11700$$
$$= 37{,}900^{\#}$$

$$\text{MIN. } V_{ci} \ @\ 20' = 1.7\ (8)\ (32.75)\ \sqrt{5000} = 31{,}500^{\#}$$

$$V_{ci} \ @\ 30' = 0.6\ (8)\ (32.75\ \sqrt{5000} + \frac{4150000}{887} + 5850$$
$$= 21{,}620^{\#}$$

$$\text{MIN. } V_{ci} \ @\ 30' = 1.7\ (8)\ (32.75)\ \sqrt{5000} = 31{,}500^{\#}$$

$$\text{MIN. } V_{ci} \ @\ 40' = 1.7\ (8)\ (32.75)\ \sqrt{5000} = 31{,}500^{\#}$$

$$f_{pc} \ @\ \text{℄} = 355 + \frac{10.1}{36}\ (1003 - 355) = 537 \text{ P.S.I.}$$

$$@\ \text{SUPPORT} = 276 + \frac{10.1}{36}\ (1209 - 276) = 538 \text{ P.S.I.}$$

$$V_p = \frac{16.75}{20\ (12)}\ (15)\ (20{,}160) = 21{,}100^{\#}$$

$d = 0.8\ (36) = 28.8''$ (MIN. "d" USED)

$$V_{cw} = b'd\ (3.5\ \sqrt{f'_c} + 0.3\ f_{pc}) + V_p$$

@ support

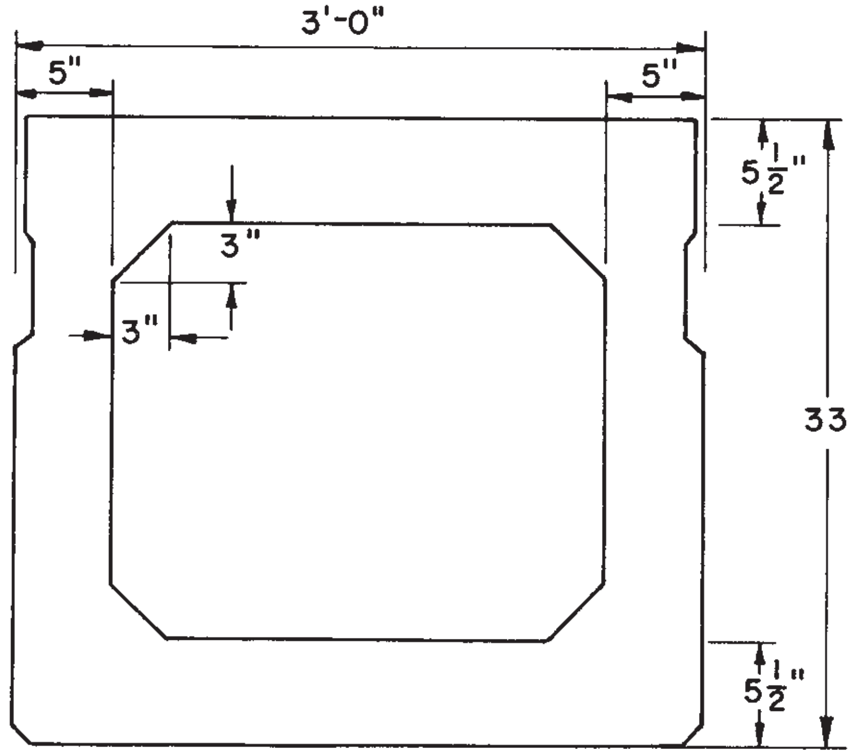
$$= (8)\ (28.8)\ \left[3.5\ (70.7) + 0.3\ (538)\right] + 21{,}100$$

$$= 115{,}000^{\#} \quad @\ \text{SUPPORT}$$

DESIGN OF THRU - VOIDED BOX GIRDER R.R. SLABS USING LIGHTWEIGHT STRUCTURAL CONCRETE

REFERENCE CRITERIA: A.R.E.A. COMMITTEE 8. ASSIGNMENT 6

STD. SPEC. FOR DESIGN AND CONSTRUCTION OF PRESTRESSED CONCRETE TRESTLES FOR RAILWAY LOADING, USING BOX BEAMS



$A = 620.5 \text{ in}^2$

$y_t = 16.71 \text{ in.}$

$y_b = 16.29 \text{ in.}$

$Z_t = 5096 \text{ in}^3$

$Z_b = 5227 \text{ in}^3$

$I = 85{,}153 \text{ in}^4$

DATA ON LIGHTWEIGHT CONCRETE :

28 - da. f'_c = 5000 P.S.I.

(DESIGN MIX FOR 25% OVERAGE OR 6250 P.S.I.)

RELEASE f'_c = 4000 P.S.I.

ASSUME THE FOLLOWING :

PLASTIC WT. OF 6250 P.S.I. CONC.	=	110	P.C.F.
28-DAY AIR-DRY WEIGHT	=	105	P.C.F.
DESIGN WEIGHT OF CONCRETE	=	110	P.C.F.

$$E_c = 33\ w^{3/2}\sqrt{f'_c}$$

$$= 33 \times 105^{3/2}\sqrt{6250} = 2.8 \times 10^6 \text{ P.S.I.}$$

NOTE: BASED ON TEST DATA FOR LIGHTWEIGHT AGGREGATE NO. 20 (NBS MONOGRAPH 74) THERE WAS ABOUT A 15% REDUCTION IN MEASURED E VS. CALCULATED E

ASSUME E_c FOR DESIGN PURPOSES = 25×10^6 P.S.I.

f'_c = 5000 P.S.I. $\qquad E_c = 2.5 \times 10^6$ P.S.I.

f'_{ci} = 4000 P.S.I. $\qquad E_c = 2.5 \times 10^6\sqrt{\frac{4}{5}} = 2.2 \times 10^6$

ALLOWABLE STRESSES:

	TEMPORARY	DESIGN
COMPRESSION:	2400 P.S.I.	2000 P.S.I.
TENSION	0	0

SUMMARY OF DESIGN LOADS AND MOMENTS:

TOTAL SUPERIMPOSED D.L. = 484 LB./FT.

BOX GIRDER D.L. = $646 \times \frac{110}{150}$ = 474 LB./FT.

DIAPHRAGMS - NONE

L.L. = E 72 IMPACT = 33.54 %

MOMENTS AT MID-SPAN:

M_S = M (SUPERIMPOSED D.L.) = $\frac{1}{8} \times 484 \times 27^2 \times 12 =$ 529,000 in.-lb.

M_G = M (BOX GIRDER D.L.) = $\frac{1}{8} \times 474 \times 27^2 \times 12 =$ 518,000 in.-lb.

$M_{LL} + I$ = 4,970,000 in.-lb.

M_T = 6,017,000 in.-lb.

ESTIMATE OF PRESTRESS LOSSES:

ASSUME A 7 % ELASTIC LOSS
AND 21 1/2 in. DIAM. STRANDS
WITH $e = 8\frac{1}{2}$ in.

INITIAL STEEL STRESS = f_{si} = 170,000 P.S.I.

$P_i = 21 \times 0.1438 \times 170,000 =$ 513,000 LB.

$$f_c \text{ @ c.g.s.} = 0.93 \times 513,000 \left(\frac{1}{620.5} + \frac{8.5^2}{85,153} \right)$$

$$= 1173 \text{ P.S.I.}$$

STRAND LOSS = $1173 \times \frac{27}{2.2}$ = 14,400 P.S.I. ($8\frac{1}{2}$ %)

relaxation = 2%

AFTER INITIAL LOSSES f_{si} = 170,000 - 14,400 - 3400 = 152,200 P.S.I.

AFTER FINAL LOSSES f_s = 170,000 - 47,000 = 123,000 P.S.I.

$$\frac{f_s}{f_{si}} = \frac{123}{152.2} = 0.808$$

STRESSES AT 1.0 h FROM END DUE TO GIRDER D.L.

$$h = 33 \text{ in.} = 2.75 \text{ ft.}$$

$$M_{10h} = 474 \times \frac{27}{2} \times 33 - 474 \times 2.75 \times \frac{33}{2}$$

$$= 211{,}000 - 21{,}500 = 189{,}500 \text{ in.-lb.}$$

$$f_t \ @ \ 1.0\,h = \frac{M_G}{Z_t} = \frac{189{,}500}{5096} = +37 \text{ P.S.I.}$$

$$f_b \ @ \ 1.0\,h = \frac{189{,}500}{5227} = -36 \text{ P.S.I.}$$

$$\text{THEN } f^{tE}_{F_iG} = 0 - 37 = -37 \text{ P.S.I.}$$

REQUIRED PRESTRESSING FORCE:

$$F = \frac{A}{h}\left[Y_b \times \frac{f_s}{f_{si}} \times f^{tE}_{FiG} + Y_t \ f^{bE}_{FG} \right] \quad \text{PCA's R/C 34 FORM. 9}$$

$$\text{BUT } f^{bE}_{FG} = f^{b}_{FT} + \frac{M_T}{Z_b} \quad \text{FORM. 10}$$

$$= 0 + \frac{6{,}017{,}000}{5227} = 1150 \text{ P.S.I.}$$

$$\text{THEN } F = \frac{620.5}{33}\left[16.29 \times 0.808 \times (-37) + 16.71 \times 1150\right]$$

$$= 352{,}500 \text{ lb.}$$

REQUIRED NO. OF STRANDS

$$N = \frac{F}{A_s f_s} = \frac{352{,}500}{0.1438 \times 123{,}000} = 20$$

SELECT 21 STRANDS
1/2 in. diam.

$$\text{THEN } F_i = 21 \times 0.1438 \times 152{,}200 = 459{,}000 \text{ lb.}$$

$$F = 21 \times 0.1438 \times 123{,}000 = 371{,}000 \text{ lb.}$$

MIN. e AT MID-SPAN:

$$f^{b}_{FT} = 0 = \frac{F}{A} + \frac{Fe}{Z_b} - \frac{M_T}{Z_b}$$

$$0 = \frac{371{,}000}{620.5} + \frac{371{,}000e}{5227} - \frac{6{,}017{,}000}{5227}$$

$$71.0\,e = 1150 - 598$$

$$e = 7.78 \text{ in.}$$

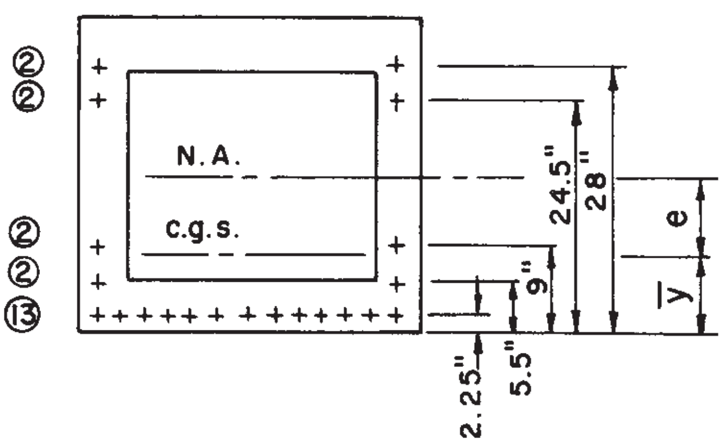
MAX. e

$$e = \frac{Z_t}{A} - \frac{Z_t}{F_i} \times f^{tE}_{FiG} \qquad \text{FORM. 11}$$

$$= \frac{5096}{620.5} - \frac{5096}{459{,}000} \times (-37)$$

$$= 8.21 + 0.41 = 8.62 \text{ in.}$$

SELECT FOLLOWING STRAND PATTERN:



Σ AREA MOMENTS:

2	x	28	=	56
2	x	24.5	=	49
2	x	9	=	18
2	x	5.5	=	11
13	x	2.25	=	29.25
				163.25

$$\bar{y} = \frac{163.25}{21} = 7.77 \text{ in.}$$

$$e = 16.29 - 7.77 = 8.52 \text{ in.}$$

SUMMATION OF STRESSES:

AT RELEASE:

$$\frac{F_i}{A} = \frac{459{,}000}{620.5} = 740 \text{ P.S.I.}$$

$$\frac{F_i e}{Z_t} = \frac{459{,}000 \times 8.57}{5096} = 767 \text{ P.S.I.}$$

$$\frac{F_i e}{Z_b} = \frac{459{,}000 \times 8.52}{5227} = 748 \text{ P.S.I.}$$

$$\frac{M_G}{Z_t} = \frac{518{,}000}{5096} = 102 \text{ P.S.I.}$$

$$\frac{M_S}{Z_t} = \frac{529{,}000}{5096} = 104 \text{ P.S.I.}$$

$$\frac{M_L}{Z_t} = \frac{4{,}970{,}000}{5096} = 975 \text{ P.S.I.}$$

1181

FINAL STRESS:

$$\frac{F}{A} = \frac{371{,}000}{620.5} = 598 \text{ P.S.I.}$$

$$\frac{Fe}{Z_t} = \frac{371{,}000 \times 8.52}{5096} = 620 \text{ P.S.I.}$$

$$\frac{Fe}{Z_b} = \frac{371{,}000 \times 8.52}{5227} = 605 \text{ P.S.I.}$$

$$\frac{M_G}{Z_b} = 99 \text{ P.S.I.}$$

$$\frac{M_S}{Z_b} = 101 \text{ P.S.I.}$$

$$\frac{M_L}{Z_b} = 951 \text{ P.S.I.}$$

1151

STAGE I AT RELEASE:

MID - SPAN

$$f^t = +740 - 767 + 102 = +75 \text{ P.S.I.}$$

$$f^b = +740 + 748 - 99 = +1389 \text{ P.S.I.}$$

AT 1.0 h FROM END

$$f^t = +740 - 767 + 37 = +10 \text{ P.S.I.}$$

$$f^b = +740 + 748 - 36 = +1452 \text{ P.S.I.}$$

STAGE III:

MID-SPAN

$$f^t = +598 - 620 + 1181 = +1159 \text{ P.S.I.}$$

$$f^b = +598 + 605 - 1151 = +52 \text{ P.S.I.}$$

ALLOW. $f_c = 2000$ WITH 0 TENSION

REQUIRED ULTIMATE MOMENT :

$$M_u = 1.892\ DL + 2.3\ (LL + I)$$

$$= 1.892 \times 1{,}047{,}000 + 2.3 \times 4{,}970{,}000$$

$$= 1{,}980{,}000 + 11{,}420{,}000 = 13{,}400{,}000 \text{ in. lb.}$$

ULTIMATE MOMENT CAPACITY :

$$A_s = 21 \times 0.1438 = 3.02 \text{ in.}^2$$

$$d = 16.71 + 8.52 = 25.23 \text{ in.}$$

$$p = \frac{3.02}{35.25 \times 25.23} = 0.0034$$

$$q = p\,\frac{f'_s}{f'_c} = 0.0034 \times \frac{250}{5} = 0.170$$

USE FIG. 2, P. 37, PCA's R/C 34

$$\frac{f'_c}{p} = \frac{5000}{0.0034} = 1.47 \times 10^6$$

$$E_{se} = \frac{f_s}{E_s} = \frac{123{,}000}{27 \times 10^6} = 0.00456$$

ASSUME $f_{su} = 240{,}000$ P.S.I.

FROM FIG. 2, CORRECTED $f_{su} = 238{,}000$ P.S.I.

$$K_u d = \frac{A_s\, f_{su}}{K_1\, K_3\, f'_c\, b}$$

$$= \frac{3.02 \times 238{,}000}{0.7 \times 5000 \times 35.25}$$

$$= 5.83 \text{ IN.}$$

N.A. IN THE WEB

$K_1 K_3 = 0.7$ IS SATISFACTORY FOR LIGHTWEIGHT CONCRETE - SEE FIG. F, p. 1326, ACI JOURNAL, DEC. 1956 PART 2

SIMILARLY $\dfrac{K_2}{K_1\, K_3} = 0.6$ IS O.K

$$M_u = A_{sr} f_{su} d \left(1 - 0.6 \frac{A_{sr}\ f_{su}}{b'd\ f'_c}\right) + 0.85 f'_c (b - b') t (d - 0.5t)$$

$$A_{sf}\ f_{su} = 0.85 f'_c (b - b') t$$

$$A_{sf} = \frac{0.85 \times 5000 \times (35.25 - 9.25) \times 5.5}{238{,}000} = 2.55 \text{ in.}$$

$$A_{sr} = 3.02 - 2.55 = 0.47 \text{ in.}$$

$$M_u = 0.47 \times 238{,}000 \times 25.23\ (1 - 0.6 \times 0.096) + 238{,}000 \times 2.55 \times (25.23 - 2.75)$$

$$= 2{,}660{,}000 + 13{,}660{,}000$$

$$= 16{,}320{,}000 \text{ in - lb.}$$

ADEQUATE CAPACITY

$$\text{APPROX. S.F.} = \frac{16.32}{6.017} = 2.7$$

SHEAR REINFORCEMENT

MAX. SHEAR AT $^1/_4$ POINT :

$$V_G = \frac{wL}{4} = \frac{474 \times 27}{4} = 3200 \text{ LB.}$$

$$V_S = 3270 \text{ LB.}$$

$$V_{LL+I} = 45{,}400 \text{ LB.}$$

$$V_T = 51{,}870 \text{ LB.}$$

$$V_{ult} = 1.892 \times 6470 + 2.3 \times 45{,}400$$

$$= 12{,}250 + 104{,}500 = 116{,}750 \text{ LB.}$$

$$V_c = 180\, b' j d = 180 \times 9.25 \times {}^7/_8 \times 25.23 = 36{,}700$$

$$V_u - V_c = 80{,}050 \text{ LB.}$$

$$A_v = {}^1/_2 \frac{(V_u - V_c)s}{f'_y\ jd} = \frac{80{,}050 \times 12}{2 \times 40{,}000 \times \frac{7}{8} \times 25.23} = 0.544 \text{ in.}^2$$

ESTIMATE OF CAMBER AND DEFLECTION:

1. PRESTRESS AT RELEASE $\Delta = \dfrac{459{,}000 \times 8.52 \times 27^2 \times 144}{8 \times 2.2 \times 10^6 \times 85{,}153} = 0.274'' \uparrow$

2. GIRDER D.L. $\Delta = \dfrac{5 \times 474 \times 27^4 \times 1728}{384 \times 2.2 \times 10^6 \times 85{,}153} = 0.030'' \downarrow$

APPROX. CAMBER = 0.244"

(ABOUT 1/4 in.)

ALL LOADS

1. PRESTRESS $\Delta = \dfrac{371{,}000 \times 8.52 \times 27^2 \times 144}{8 \times 2.5 \times 10^6 \times 85{,}153} = 0.195'' \uparrow$

2. DL + LL $\Delta = \dfrac{5 \times 5508 \times 27^4 \times 1728}{384 \times 2.5 \times 10^6 \times 85{,}153} = 0.310'' \downarrow$

484
474
4550
5508

APPROX. DEFLECTION = 0.115" $\downarrow$

(ABOUT 1/8 in.)

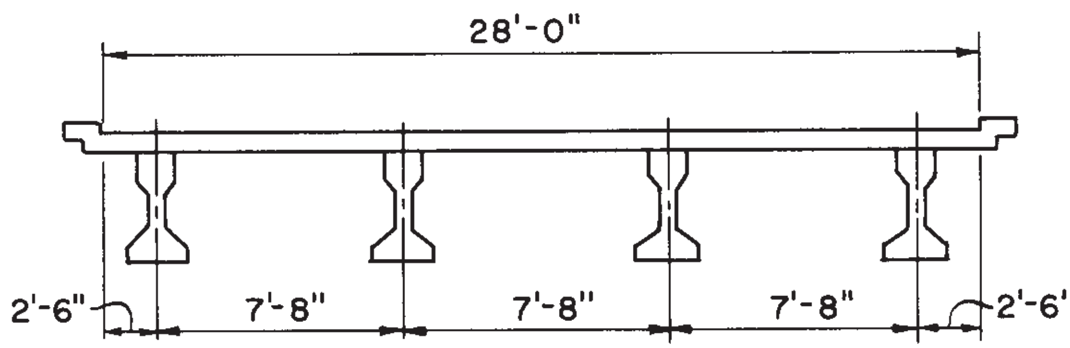
---

3. DL ONLY $\Delta = \dfrac{5 \times 958 \times 27^4 \times 1728}{384 \times 2.5 \times 10^6 \times 85{,}153} = 0.054'' \downarrow$

APPROX. RESIDUAL CAMBER = 0.141" $\uparrow$

EXAMPLE

PRESTRESSED CONCRETE BRIDGE GIRDER



BASIC DATA

SPAN : 75'-0"

WIDTH : 28'-0" (CURB TO CURB)

ACI - ASCE 323 REPORT-AASHO BRIDGE SPECS.

f_{ci} = 4000 PSI

f'_c = 5000 PSI (GIRDER) - 4000 PSI (DECK)

$$\left.\begin{array}{l} E_c \text{ (GIRDER)} : 4.5 \times 10^6 \text{ PSI} \\ E_c \text{ (SLAB)} : 3.5 \times 10^6 \text{ PSI} \end{array}\right\} \text{BY TEST}$$

SIMPLE SPAN

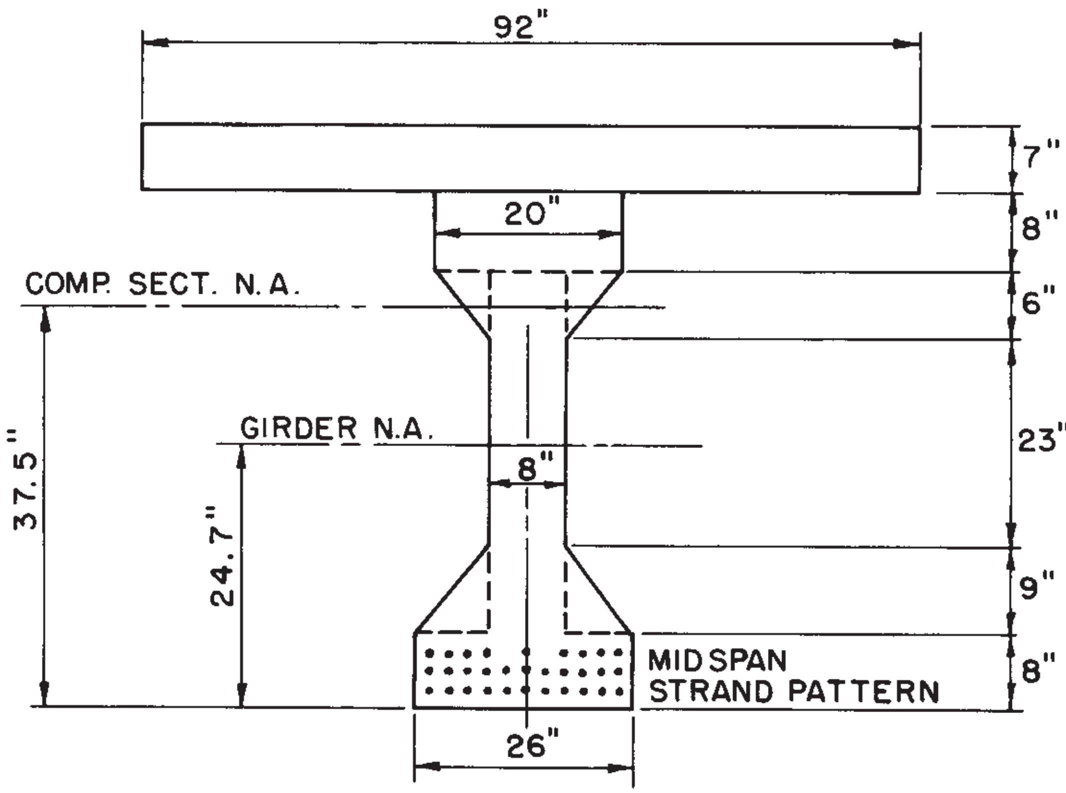
SLAB THICKNESS = 7 IN.

IMPACT : 25 %

USE TYPE IV AASHO GIRDER

H20 - S16 LOADING

DESIGN FOR INTERIOR GIRDER



SECTION PROPERTIES

SECTION	A	y	Ay	d	Ad²		Io
20 x 8	160	50	8000	25.3	102500	20 x 8 ³/12	850
6 x 6	36	44	1584	19.3	13400	6 x 6 ³/18	70
38 x 8	304	27	8200	2.3	1600	8 x 38 ³/12	36500
9 x 9	81	11	890	13.7	15200	9 x 9 ³/18	360
26 x 8	208	4	832	20.7	89000	26 x 8 ³/12	1110
	789		19506	12.8	221700		38890
					129000		
* $\frac{3.5}{4.5}$ x 92 x 7	501	575	28800	20	200000	3.5/4.5 x 92 x 7 ³/12	2050
	1290		48306		550700		40940

$$y_b = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{19500}{789}$$

= 24.7 IN. (GIRDER ALONE)

$$y_b^* = \frac{48300}{1290}$$

= 37.5 IN. (COMP. SECT.)

* CORRECTION FOR DIFFERENT E VALUES GIRDER AND TOPPING

$$I_{xx} = 221700 + 38900 = 260600 \text{ IN.}^4 \text{ (GIRDER)}$$

$$I_{xx} = 550700 + 40900 = 591600 \text{ IN.}^4 \text{ (COMP. SECT.)}$$

$$\left.\begin{aligned} S_{bG} &= \frac{260600}{24.7} = 10550 \text{ IN.}^3 \\ S_{tG} &= \frac{260600}{29.3} = 8900 \text{ IN.}^3 \end{aligned}\right\} \text{GIRDER}$$

$$\left.\begin{aligned} S_{bc} &= \frac{591600}{37.5} = 15750 \text{ IN.}^3 \\ S_{tc} &= \frac{591600}{23.5} = 25200 \text{ IN}^3 \end{aligned}\right\} \text{COMP. SECT.}$$

DESIGN MOMENTS

GIRDER ALONE

$$M_D = \left(\frac{789}{144}\right)(.150)(75)^2(1.5) = 6930 \text{ IN.}^K$$

GIRDER PLUS TOPPING (ADD 1/2" FOR WEAR. SURF.)

$$M_D = \left(\frac{789 + 7.5 \times 92}{144}\right)(.150)(75)^2(1.5)$$

$$= 13000 \text{ IN.}^K$$

LIVE LOAD

$$M_L = 1075 \times 1.25 \times \frac{7.65}{5.5} \times 1/2 \times 12 \quad \text{(AASHO)}$$

$$= 11200 \text{ IN.}^K$$

DETERMINATION OF PRESTRESS FORCE

EST. $e = 21''$

FINAL BOTTOM FIBER STRESS UNDER FULL LOAD.

$$f_{bf} = \frac{M_D}{S_{bG}} + \frac{M_L}{S_{bc}}$$

$$= \frac{13000}{10.55} + \frac{11200}{15.75}$$

$$= 1235 + 710$$

$$= 1945 \text{ P.S.I.}$$

$$\frac{S_b}{A} = \frac{10550}{789} = 13.4$$

$$F_i = \frac{A \times f_{bf}}{0.8\left(1 + \frac{eA}{S_b}\right)}$$

$$= \frac{(790)(1.945)}{0.8\,(1 + 1.57)}$$

$$= 750^{K}$$

NO. STRANDS (USE $\frac{1}{2}''$ STRANDS $A_s = 0.1435$)

$$\frac{750}{25.3^{*}} = 30 \quad (31)$$

$$e = 24.7 - \left[\frac{11 \times 2 + 11 \times 4 + 8 \times 6}{31}\right]$$

$$= 20.9'' \quad \text{(ADD ONE STRAND)}$$

$$F_i = 31 \times 25.3$$

$$= 785^{K}$$

$$e = 24.7 - \left[\frac{11 \times 2 + 11 \times 4 + 9 \times 6}{32}\right]$$

$$= 20.9''$$

* INITIAL STRESSING FORCE FOR $\frac{1}{2}''$ STRAND

DESIGN STRESSES

INITIAL (MIDSPAN)

BOTTOM, $f_{bi} = \dfrac{(.95)(785)}{0.789} + \dfrac{(.95)(785)(20.9)}{10.55} - \dfrac{6930}{10.55}$

$= 945 + 1480 - 655$

$= 1770 < 2400$ P.S.I. ($0.6 f_{ci}$ - ACI 323 207.3.1)

TOP $f_{ti} = 945 - \dfrac{(0.95)(785)(20.9)}{8.90} - \dfrac{6930}{8.90}$

$= 945 - 1750 + 780$

$= -25 < 190$ P.S.I. ($3\sqrt{f'_{ci}}$ - ACI 323 207.3.1)

INITIAL (END)

BOTTOM $f_{bi} = 945 + 1480 = 2425 > 2400$ P.S.I.

TOP $f_{ti} = 945 - 1750 = -805 > 190$ P.S.I.

SOME STRANDS MUST BE DEFLECTED OR UNBONDED TO REDUCE THE END STRESSES: USE UNBONDING METHOD.

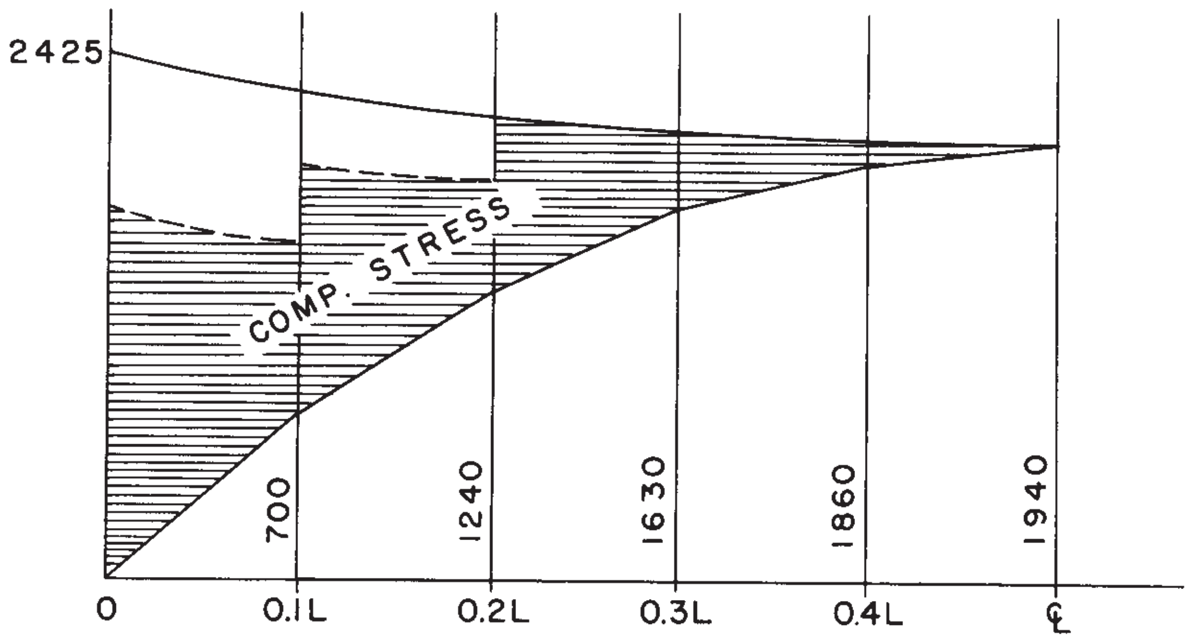
FINAL (MIDSPAN)

BOTTOM $f_{bf} = \dfrac{(0.8)(785)}{0.789} + \dfrac{(0.8)(785)(20.9)}{10.55} + \dfrac{13000}{10.55} + \dfrac{11200}{15.75}$

$= 800 + 1245 - 1230 - 710 = 90 > 0$ P.S.I.

TOP (GIRDER) $f_{tf} = 800 - \dfrac{(0.8)(785)(20.9)}{8.9} + \dfrac{13000}{8.9} + \dfrac{(11200)(16.5)}{591.6}$

$= 1090 < 2000$ P.S.I. ($0.4 f'_c$ - ACI 323 207.3.2)



1/2 MOMENT DIAGRAM IN TERMS OF BOTTOM FIBER STRESS.

ALLOW FOR STRESS TRANSFER LENGTH OF 60"

TOTAL NO. OF STRANDS = 31

UNBOND $31 - \frac{1940}{2425} \times 31$ = SAY 8 $\qquad$ (0.1)L = 7.5'

UNBOND 4 $\quad$ (0.2) L = 15'

ALL OTHERS FULLY BONDED.

ULTIMATE MOMENT

$$p \text{ (COMP.)} = \frac{4.45}{92 \times 56.5} = 0.00086$$

$$f_{su} = (250000)\left(1 - \frac{1}{2}\frac{250000}{4000} \times 0.00086\right) = 236000 \text{ ACI 323 209.2.2}$$

$$q = \frac{236000}{4000} \times 0.00086 = 0.051$$

$$M_u = (92)(56.5)^2(4000)(0.051)(1 - 0.6 \times 0.051)$$

$$= 58500 \text{ IN.}^K > 47500 \ (1.5D + 2.5L)$$

SHEAR

V MAX @ 1/4 SPAN: SEC. 210.2.5 (ACI-ASCE 323)

$$D.L. = \left[\frac{789 + (7.5 \times 92)}{144}\right](150)\ \frac{(75)}{4}$$

$$= 28.8^K$$

$$L.L. = \frac{(32)(56.25)+(32)(42.25)+(8)(28.25)}{75}$$

$$= 45.0^K \quad \text{(LANE LOAD)}$$

$$(45)(1.25)\left(\frac{7.65}{5.5}\right)\left(\frac{1}{2}\right) = \underline{40}^K \quad \text{(GIRDER LOAD)}$$

$$V_u = (1.5 \times 28.8) + (2.5)(40) \qquad \text{SEC. 205.3.2}$$

$$= 143.2^K$$

$$V_c = (180)(8)(56.5)(1 - 0.6 \times 0.051) \qquad \text{SEC. 210.2.2}$$

$$= 79^K$$

$$A_v = 1/2\ \frac{(143.2 - 79)\,S}{40 \times 0.97 \times 56.5} \qquad \text{SEC. 210.2.2}$$

ASSUME #4 $\qquad A_v = 0.4\ \text{IN.}^2$

$$S = 27''$$

$$\text{MIN. } A_v = 0.0025 \times 8 \times 27 \qquad \text{SEC. 210.2.3}$$

$$= 0.54\ \text{IN.}^2 > 0.4$$

DECREASE SPACING TO 20"

MAX. S $= 3/4 \times 56.5 = 42.3"$ SEC. 210.2.4

USE #4  THROUGH OUT LENGTH OF MEMBER @ 20"

COMPOSITE DESIGN SEC. 212.3.2

SHEAR BETWEEN COMPOSITE SECTION AND GIRDER

$$v = \frac{V_u Q}{I_c t'}$$

$t = 20"$

$I = 591000$ IN.^{4}

$Q = 10000$

$V = 40000$ #

$Q = 0.778 \times 7 \times 92\ (23.5 - 3.5)$

$Q = 10000$ IN.^{3}

$$v = \frac{2.5 \times 40000 \times 10000}{591000 \times 20}$$

$v = 85$ P.S.I. < 150 SEC. 212.3.3

IT REQUIRES THE MINIMUM STEEL TIE REQUIREMENTS OF SEC. 212.3.4 ALSO REQUIRES CONTACT SURFACE TO BE ARTIFICIALLY ROUGHENED.

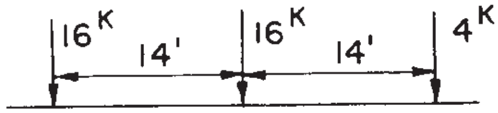
MINIMUM REQUIRED = 0.22 SQ. IN./FT. SEC. 212.3.4

AREA STIRRUPS #4 $= \dfrac{0.392 \times 12}{20} = .236\ \square''/\text{FT.}$

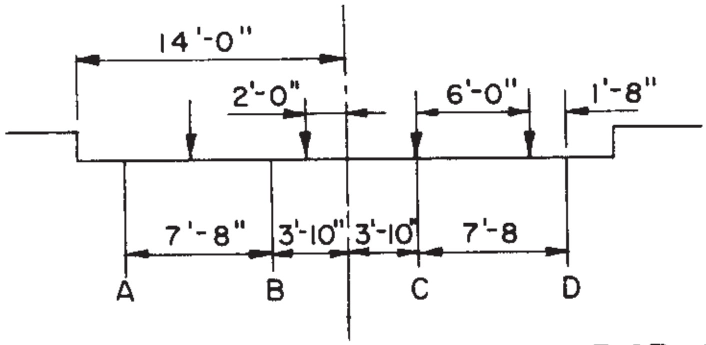
USE STIRRUPS FOR TIES

SHEAR INVESTIGATION (IN ACCORDANCE WITH ACI 318-63 CODE)

(a) LOADING: LIVE LOAD H 20 - S16



LONGITUDINAL SPACING OF WHEEL LOADS



TRANSVERSE DISTRIBUTION IN ACCORDANCE WITH AASHO, SECT. 3, 1.3.1 (a)

LOAD ON GIRDER "C": $P = \dfrac{7.67 + 1.67 + 1.83}{7.67} = 1.46$

LOAD IN ACCORDANCE WITH 1.3.1 (b): $P = \dfrac{7.67}{5.5} = 1.40$

CHOOSE $P = 1.46$ FOR 1^{ST} WHEEL AND $P = 1.40$ FOR THE OTHER TWO

DEAD LOAD: GIRDER $\dfrac{789}{144}\ 0.150 = 0.82$

SLAB & SURFACE $\dfrac{7.5 \times 92}{144}\ 0.150 = \underline{0.72}$

1.54 KIPS/FT.

(b) SHEAR Vu *

AT SUPPORT:

LIVE LOAD $\dfrac{1.25}{75}(1.46 \times 75 \times 16 + 1.40 \times 61 \times 16 + 1.40 \times 47 \times 4) = 56.5^K$

DEAD LOAD $\tfrac{1}{2} \times 75 \times 1.54 = 57.8^K$

$Vu = 1.5 \times 57.8 + 2.5 \times 56.5 = 227.7^K$

AT MIDSPAN: $\dfrac{1.25}{75}(1.46 \times 37.5 \times 16 + 1.40 \times 23.5 \times 16 + 1.40 \times 9.5 \times 4) = 24.2^K$

$Vu = 2.5 \times 24.2 = 60.5^K$

* THE REDUCTION FACTOR, ϕ, IS OMITTED IN SUBSEQUENT COMPUTATIONS TO COMPENSATE FOR THE HIGHER LOAD FACTOR FOR HIGHWAY LIVE LOADS.

(C) MINIMUM WEB REINFORCEMENT AND V_s

EFFECTIVE DEPTH, $d = y_t + e + t =$

$$29.3 + 20.8 + 7.0 = 57.1 \text{ IN.}$$

$b' = 8$ IN. $\quad s = 12$ IN. $\quad f'_s = 250^K/\text{SQ. IN.}$

$A_s = 32 \times 0.1435 = 4.59$ SQ. IN. $\quad f_y = 40^K/\text{SQ. IN.}$

$$A_v = \frac{A_s}{80} \frac{f'_s}{f_y} \frac{s}{d} \sqrt{\frac{d}{b'}}$$

$$= \frac{4.59}{80} \frac{250}{40} \frac{12}{57.1} \sqrt{\frac{57.1}{8}} = \underline{0.20 \text{ SQ. IN./FT.}}$$

$$V_s = \frac{f'_s}{80} A_s \sqrt{\frac{d}{b'}}$$

$$\frac{250}{80} \, 4.59 \sqrt{\frac{57.1}{8}} = \underline{38.4^K}$$

(d) V_{ci} min :

$$V_{ci} \text{ min} = 1.7 \, b'd \sqrt{f'_c}$$

$$= 1.7 \times 8 \times 57.1 \times \sqrt{5000} = \underline{54.9^K}$$

(e) CAPACITY OF SECTIONS

$$\boxed{X = d/2 \cong 2 \text{ FT. FROM SUPPORT}}$$

$$V_u = 60.5 + \frac{35.5}{37.5}(227.7 - 60.5) = 219.0^K$$

$$V_{cw} = b'd \, (3.5\sqrt{f'_c} + 0.3 \, f_{pc}) + V_p$$

d^*: THE GREATER OF $0.80 \, (54.0 + 7.0) = 48.8$ IN.

OR $\underline{d = 57.1 \text{ IN.}}$ ⟵ CHOSEN

*EFFECT OF UNBONDED TENDONS ON "d" IS NEGLECTED

$$f_{pc} = \frac{F}{A} - \frac{Fe}{I}(y_b^c - y_b) + \frac{M_D}{I}(y_b^c - y_b)$$

$$F = (32-8)\ 0.1435 \times 0.80 \times 175 = 482^K$$

$$M_D = 1.54\ \frac{2\ (75-2)}{2} = 112\ '\text{-}K$$

$$f_{pc} = \frac{482}{.789} - \frac{482 \times 20.8}{260.6}(37.5 - 24.7) + \frac{112 \times 12}{260.6}(37.5 - 24.7)$$

$$= 184 \text{ P.S.I.}$$

$$V_{cw} = 8 \times 57.1\ (3.5\sqrt{5000} + 0.3 \times 184)\ \frac{1}{1000} = \underline{138.5^K}$$

$$V_{ci} = \infty \text{, BECAUSE } \frac{M}{V} = \frac{d}{2}$$

$$A_v = \frac{V_u - V_{cw}}{d\ f_y} S$$

$$= \frac{(219.0 - 138.5)}{57.1 \times 40} 12 \qquad = \underline{0.42 \text{ SQ. IN. /FT.}}$$

$$\boxed{X = 0.2\ L = 15 \text{ FT. FROM SUPPORT}}$$

$$\left.\begin{array}{l} F(x < 0.2L) = (32-4)\ 0.1435 \times .80 \times 175 = 562^K \\ F(x > 0.2L) = 32 \times 0.1435 \times .80 \times 175 = 643^K \end{array}\right\} \text{THE EFFECT OF THE TRANSFER LENGTH IS NEGLECTED.}$$

$$M_D = 1.54\ \frac{15\ (75-15)}{2} = 693\ '\text{-}K$$

$$f_{pc}\ \underline{(X < 0.2L)} = \frac{562}{.789} - \frac{562 \times 20.8}{260.6}(37.5 - 24.7) + \frac{693 \times 12}{260.6}(37.5 - 24.7)$$

$$= 546 \text{ P.S.I.}$$

$$V_{cw} = 8 \times 57.1\ (3.5\sqrt{5000} + 0.3 \times 546)\ \frac{1}{1000} = 187.7^K$$

$$f_{pc}\ \underline{(X > 0.2L)} = \frac{643}{.789} - \frac{643 \times 20.8}{260.6}(37.5 - 24.7) + \frac{693 \times 12}{260.6}(37.5 - 24.7)$$

$$= 566 \text{ P.S.I.}$$

$$V_{cw} = 8 \times 57.1\ (3.5\sqrt{5000} + 0.3 \times 566)\ \frac{1}{1000} = 190.3^K$$

$$V_{ci} = 0.6\, b'd\sqrt{f'_c} + \frac{M_{cr}}{\frac{M}{V} - \frac{d}{2}} + V_d$$

$d = 57.1$ IN. (AS BEFORE)

$V_d = 0.3 \times 75 \times 1.54 = 34.6^K$

$\frac{M}{V} = 0.2\,L = 15.0'$

$\frac{M}{V} - \frac{d}{2} = 15.0 - \frac{57.1}{12 \times 2} = 12.6'$

$$M_{cr} = \frac{I^c}{y_b^c}\left(6\sqrt{f'_c} + f_{pe} - f_d\right)$$

$f_d = \frac{M_D}{S_b} = \frac{693 \times 12}{10.55} = 790$ P.S.I.

$f_{pe}\ (X < 0.2L) = \frac{562}{.789} + \frac{562 \times 20.8}{10.55} = 1820$ P.S.I.

$$V_{ci} = 0.6 \times 8 \times 57.1 \times \sqrt{5000}\ \frac{1}{1000} + \frac{1}{12.6}\ \frac{591{,}600}{37.5 \times 12}\left(6\sqrt{5000} + 1820 - 790\right) + 34.6$$

$= 19.4 + 152 + 34.6 = 206.0^K$

$f_{pe}\ (X > 0.2L) = \frac{643}{.789} + \frac{643 \times 20.8}{10.55} = 2082$ P.S.I.

$$V_{ci} = 19.4 + \frac{591.600}{12.6 \times 37.5 \times 12}\left(6\sqrt{5000} + 2082 - 790\right) + 34.6$$

$= 19.4 + 179.3 + 34.6 = 233.3$

$X < 0.2\,L$: $V_{cw} = 187.7^K$

$X > 0.2\,L$: $V_{cw} = 190.3^K$

$X < 0.2\,L$: $V_{ci} = 206.0^K$

$X > 0.2\,L$: $V_{ci} = 233.3^K$

THESE VALUES ARE PLOTTED ON THE STRESS DIAGRAM. THE VALUES FOR OTHER SECTIONS ALONG THE SPAN WERE DERIVED SIMILARLY.

f. WEB REINFORCEMENT

FROM FACE OF SUPPORT TO $0.1L = 7.5$ FT. : A_v REQUIRED $= 0.42^{\square''}/1$

PROVIDE #6 ⊔ STIRRUPS @ 24 IN. O.C. = 0.44 SQ.IN. / FT.

FOR REMAINDER OF SPAN, PROVIDE #4 STIRRUPS @ 24 IN. O.C. = 0.20 SQ.IN./FT.

$A_v(\text{REQUIRED}) = A_v(\text{MIN.}) = 0.20$ SQ.IN./FT.

BOND AT CONTACT SURFACE (CHAPTER 25)

$A_v(\text{MIN.}) = 0.0015 \times 20 \times 12 = 0.36$ SQ. IN. / FT. VERT. TIES

$$V \text{ (LIVE LOAD ONLY)} = \frac{1.25}{75}\left[1.46 \times 16 \times 67.5 + 1.40\,(16 \times 53.5 + 4 \times 39.5)\right]$$

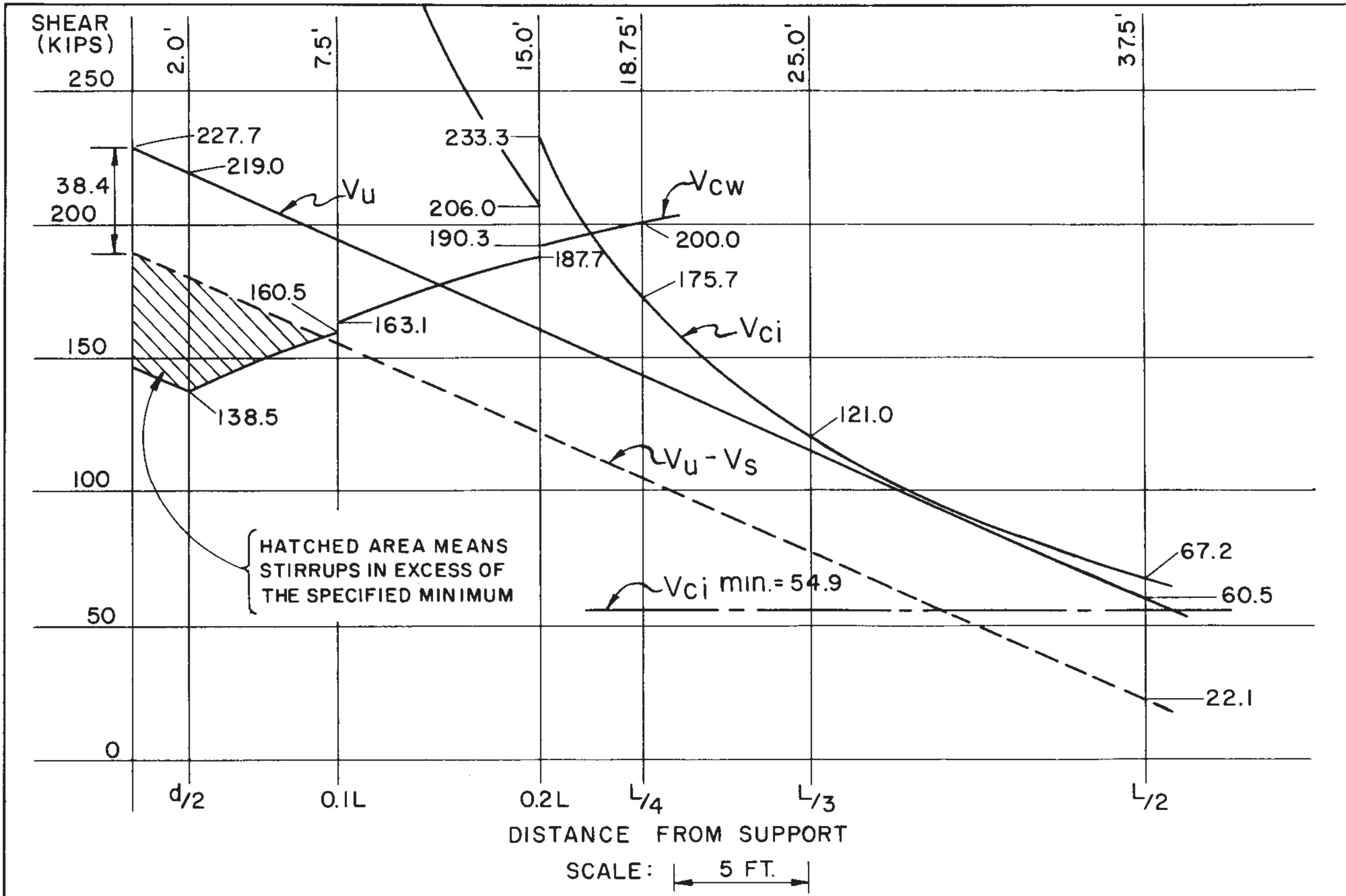
$$@\ X = 0.1L \qquad = 50.0^{K}$$

$$V_h = \frac{V\,Q}{I\,b'} \qquad b' = 20 \text{ IN.} \qquad I = I^{c} = 591600 \text{ IN.}^4$$

$$Q = 0.778 \times 7 \times 92\,(23.5 - 3.5) = 10030 \text{ IN.}^3$$

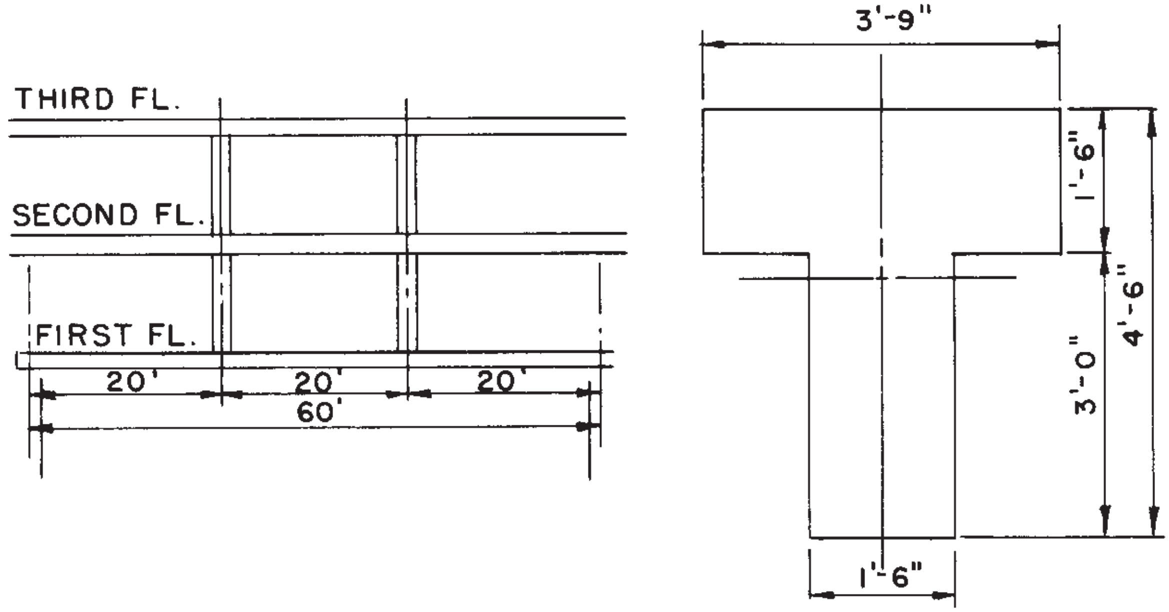
$$V_h = \frac{50000 \times 10030}{591600 \times 20} = 42.4 \text{ P.S.I.} > 40 \text{ P.S.I.}$$

V_h IS LESS THAN 40 P.S.I. AT $X = 12$ FT. FROM THERE ON BOND REQUIREMENTS ARE SATISFIED WITH OUT TIES, PROVIDED THE SURFACE IS ROUGH AND CLEAN. EXTRA TIES IN THE AMOUNT OF $(0.36 - 0.20) = 0.16$ SQ. IN. / FT. MUST BE PROVIDED BETWEEN $X = 0.1L = 7.5$ FT. AND $X = 12$ FT. NO EXTRA TIES ARE REQUIRED BETWEEN THE SUPPORT AND $X = 0.1L$, BECAUSE THE WEB REINFORCEMENT, $A_v = 0.44$ SQ. IN. / FT. EXCEEDS THE MINIMUM FOR VERTICAL TIES (0.36 SQ. IN. / FT.). ALL STIRRUPS ARE ASSUMED TO EXTEND INTO THE COMPOSITE SLAB.



SHEAR DIAGRAM

POST - TENSIONED BEAM



DESIGN THE ABOVE FIRST FLOOR BEAM FOR THE FOLLOWING LOAD CONDITIONS. ALSO CONTROL DEFLECTIONS TO PREVENT DAMAGE ABOVE

LOADS

			TOTALS	
FIRST FLOOR:				
D.L.	GIRDER:	1.52 K / FT.		
	FLOOR, FINISH, ETC.	0.98	2.50	K / FT.
L.L.		1.50	1.50	
SECOND FLOOR:				
D.L.	(PER COLUMN)		40.0	K
L.L.	(PER COLUMN)		20.0	K
THIRD FLOOR:				
D.L.	(PER COLUMN)		40.0	K
L.L.	(PER COLUMN)		20.0	K

PHSICAL PROPERTIES

$f'_c = 5000$ P.S.I.

$f'_{ci} = 4000$ P.S.I.

$f'_s = 160000$ P.S.I.

NORMAL WEIGHT CONCRETE

SECTION PROPERTIES:

	A	y	Ay	I	
3.75 x 1.5	5.62	+0.75	+4.22	$3.75 \times \frac{(1.5)^3}{3} =$	4.21
1.5 x 3.0	4.50	-1.50	-6.75	$1.5 \times \frac{(3.0)^3}{3} =$	13.50
	10.12		-2.53		17.71

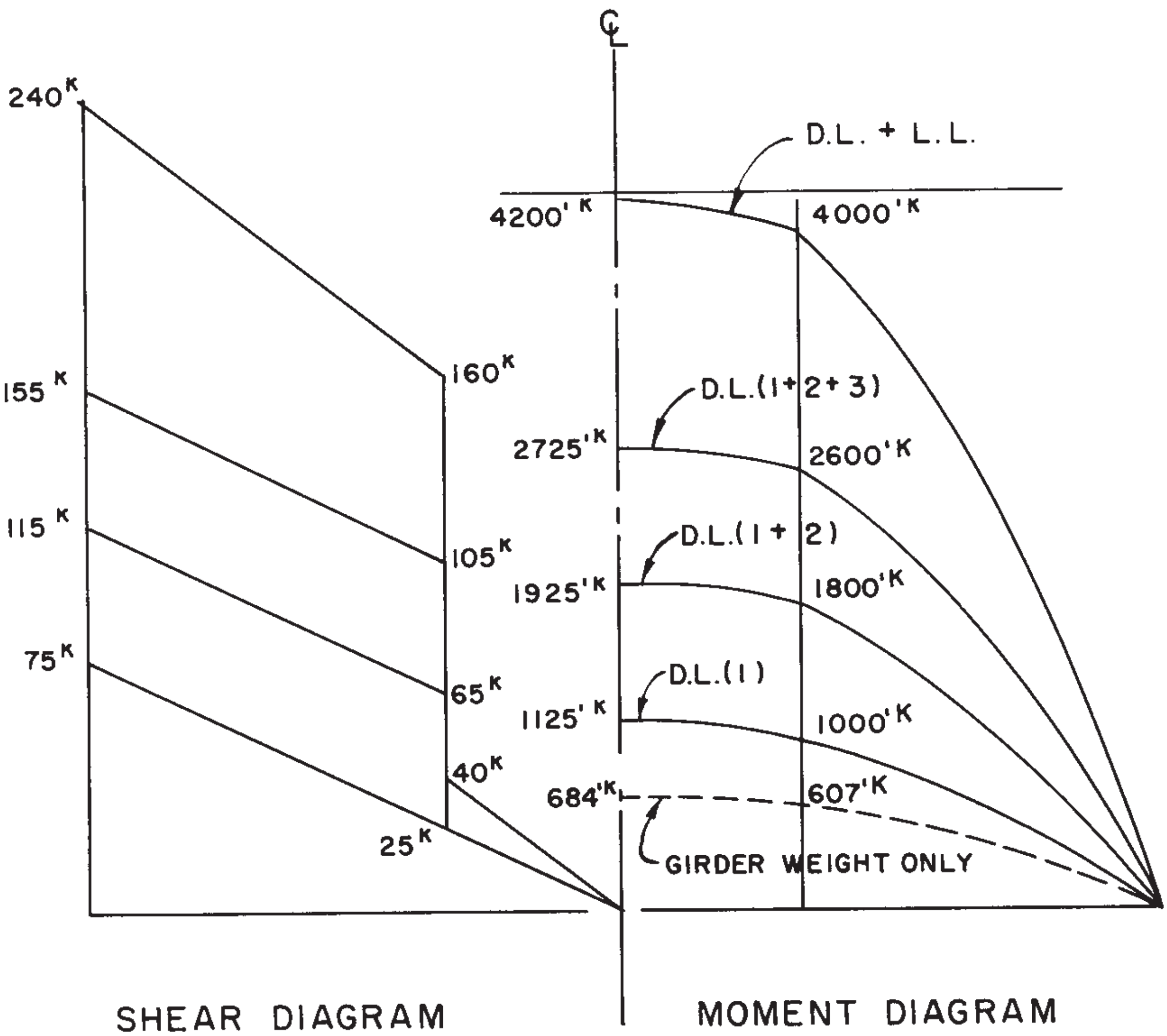
$$\bar{y} = \frac{-2.53}{10.12} = -0.25 \text{ FT.} = -3''$$

$$-10.12\,(0.25)^2 = -0.63$$

$$I = 17.08 \text{ FT.}^4$$

$C_T = 1.75$ $\quad S_T = 9.75 \text{ FT}^3$

$C_b = 2.75$ $\quad S_B = 6.20 \text{ FT}^3$


240K
160K
155K
105K
115K
65K
75K
40K
25K
SHEAR DIAGRAM
℄
D.L. + L.L.
4200'K
4000'K
D.L.(1+2+3)
2725'K
2600'K
D.L.(1 + 2)
1925'K
1800'K
D.L.(1)
1125'K
1000'K
684'K
607'K
GIRDER WEIGHT ONLY
MOMENT DIAGRAM

ESTIMATE FINAL PRESTRESSING FORCE

DESIGN FOR ZERO TENSION UNDER MAXIMUM LOAD.

$$0 = \frac{F_F}{A} + \frac{F_F \times e}{S_B} - \frac{M_T}{S_B}$$ (+DENOTES COMPRESSION)

WHERE F_F = TOTAL FINAL PRESTRESS FORCE AFTER ALL LOSSES

A = AREA OF UNCRACKED SECTION

M_T = MAXIMUM TOTAL MOMENT (D.L. + L.L.)

e = ECCENTRICITY OF APPLICATION OF PRESTRESS FORCE

ASSUME $e = 33 - 8 = 25''$

S_B = BOTTOM FIBER SECTION MODULUS

$$0 = \frac{F_F}{10.12 \times 144} + \frac{F_F \times 25}{6.20 \times (12)^3} - \frac{4200 \times 12}{6.20 \times (12)^3}$$

F_F = 1560 KIPS

TRY 12 STRESSTEEL BARS $1\frac{3}{8}\phi$*

f'_s = 160000 P.S.I.

$f_{si} = 0.7 \times f'_s$ = 112000 P.S.I.

ESTIMATED LOSSES = 25000 P.S.I. (THIS VALUE SHOULD BE CHECKED LATER)

f_{sf} = 87000 P.S.I.

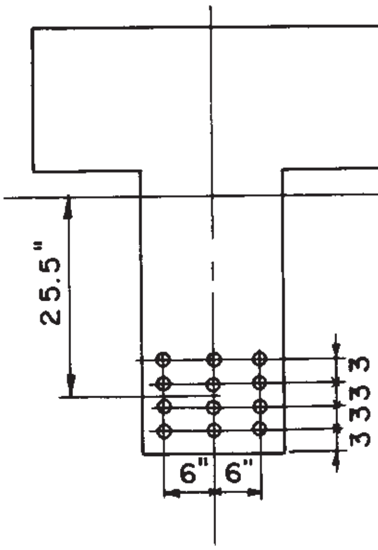
A_s = 12 x 1.485 = 17.83 SQ. IN.

F_F = 17.83 x 87 = 1550 KIPS

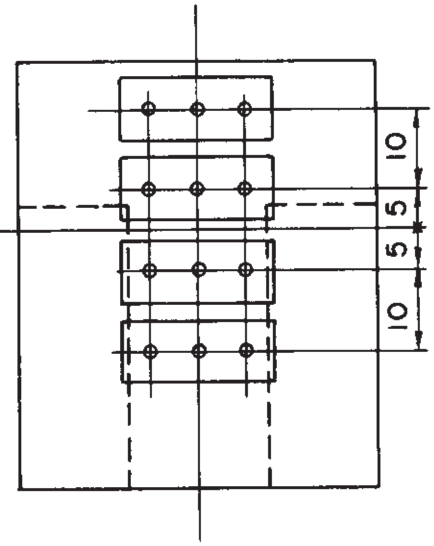
*NOTE:

THERE ARE SEVERAL OTHER POST-TENSIONING SYSTEMS WHICH CAN BE USED AS APPROPRIATE PRE-STRESSING STEEL REINFORCEMENT.

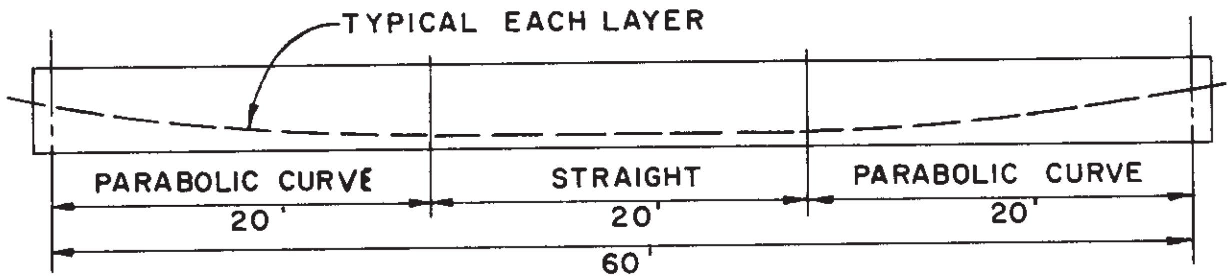
TENDON PATTERN:



SECTION @ ℄



SECTION @ END OF BEAM



ELEVATION

NOTE:

IN POST-TENSIONED MEMBERS, THE CHOICE OF TENDON PATTERN HAS ALMOST UNLIMITED FLEXIBILITY, DEPENDING ON HOW COMPLICATED THE MOMENT DIAGRAM IS.

ANALYSIS OF STRESSES

DUE TO:	AT ℄ (X = 30') TOP	BOTTOM	AT COL. LINE (X = 20') TOP	BOTTOM	AT X = 10' TOP	BOTTOM
D.L. (1)	+ 800	- 1264	+ 711	- 1122	+ 444	- 701
D.L. (2)	+ 569	- 897	+ 569	- 897	+ 284	- 448
D.L. (3)	+ 569	- 897	+ 569	- 897	+ 284	- 448
L.L.	+ 1050	- 1655	+ 997	- 1570	+ 551	- 870
TOTAL PRESTRESSING FINAL FORCE:						
$\frac{F_F}{A}$	+ 1062	+ 1062	+ 1062	+ 1062	+ 1062	+ 1062
$\frac{F_F e}{S}$	- 2340	+ 3690	- 2340	+ 3690	- 1755	+ 2765
	- 1278	+ 4752	- 1278	+ 4752	- 693	+ 3827

NOTE:

AN INSPECTION OF THE ABOVE TABLE WILL SHOW THAT A BEAM OF THESE DIMENSIONS COULD NOT HAVE BEEN MANUFACTURED AS A PRETENSIONED MEMBER. BOTH THE TENSILE STRESS AT THE TOP FIBER AND THE COMPRESSIVE STRESS AT THE BOTTOM FIBER WOULD HAVE BEEN TOO HIGH.

BY MULTIPLE STAGE POST-TENSIONING (SEE NEXT PAGE) STRESSES CAN BE MAINTAINED AT ALL TIMES WITHIN ALLOWABLE VALUES.

	AT ℄ (X=30')		AT COL. LINE (X=20')		AT X=10'	
	TOP	BOTTOM	TOP	BOTTOM	TOP	BOTTOM
D.L.(1)	+ 800	-1264	+ 711	- 1122	+444	- 701
1st P/T(50%)	- 639	+2376	- 639	+2376	-346	+1914
	+ 161	+1112	+ 72	+1254	+ 98	+1213
D.L.(2)	+ 569	- 897	+ 569	- 897	+284	- 448
	+ 730	+215	+ 641	+357	+382	+765
2nd P/T(33%)	- 426	+1584	- 426	+1584	-231	+1276
	+ 304	+1799	+ 215	+1941	+151	+2041
D.L.(3)	+ 569	-897	+ 569	-897	+284	- 448
	+ 873	+902	+ 784	+1044	+435	+1593
3rd P/T(17%)	- 213	+792	- 213	+792	-116	+638
	+ 660	+1694	+ 571	+1836	+319	* +2231
L.L.	+ 1050	-1655	+ 997	-1570	+551	-870
	+ 1710	** +39	+ 1568	+266	+870	+1361

* MAX. ALLOWABLE: 0.45 x 5000 = 2250 P.S.I.

** MIN. ALLOWABLE: ZERO STRESS

NOTE:

EVIDENTLY, A SLIGHT CHANGE IN THE TENDON PATTERN, BY SHORTENING THE CENTRAL STRAIGHT PORTION, WOULD REDUCE THE VALUE OF + 2231 P.S.I. OBTAINED FOR THE BOTTOM FIBER AT THE 1/6 POINT. (X = 10')

ANALYSIS OF DEFLECTIONS*:

DUE TO:	AT ℄	AT COL. LINES
D.L. (1)	+ 0.510 IN.	+ 0.442 IN.
FIRST P/T (50%)	- 0.827	- 0.728
	- 0.317	- 0.286
D.L. (2)	+ 0.371	+ 0.323
	+ 0.054	+ 0.037
SECOND P/T (33%)	- 0.552	- 0.485
	- 0.498	- 0.448
D.L. (3)	+ 0.371	+ 0.323
	- 0.127	- 0.125
THIRD P/T (17%)	- 0.276	- 0.243
	- 0.403	- 0.368
L.L.	+ 0.677	+ 0.588
	+ 0.274	+ 0.220

NOTE: (+) DENOTES DOWNWARD DEFLECTIONS

* DEFLECTIONS COMPUTED BY:

$$\Delta = \int \frac{Mx}{EI}\, dx$$

WHERE $E = w^{1.5}\; 33\sqrt{f'_c}$ $(w = 145)$

I = MOMENT OF INERTIA OF THE SOLID CONCRETE SECTION.

REMARKS WITH RESPECT TO DEFLECTIONS:

FIRST FLOOR:

AFTER THE FIRST FLOOR IS CAST AND INITIALLY POST-TENSIONED THE MAXIMUM DEFLECTION AT ℄ VARIES FROM -0.181 TO +0.591 INCHES.

SECOND FLOOR:

AFTER THE SECOND FLOOR IS CAST, THE MAXIMUM DEFLECTION AT THE COLUMN LINES (EQUIVALENT TO A SETTLEMENT OF THE COLUMN) VARIES FROM -0.485 TO +0.183 INCHES.

THIRD FLOOR:

AFTER THE THIRD FLOOR IS CAST, THE MAXIMUM DEFLECTION AT THE COLUMN LINES VARIES FROM -0.243 TO + 0.345 INCHES.

DEFLECTIONS DUE TO LIVE LOADS:

THE MAXIMUM DEFLECTION DUE TO LIVE LOAD IS 0.677 INCHES OR $^{1}/_{1065}$ OF THE SPAN.

EFFECT OF TIME ON DEFLECTIONS:

FOR SIMPLICITY OF PRESENTATION, NO CONSIDERATION HAS BEEN GIVEN TO THE EFFECT OF TIME IN COMPUTING DEFLECTIONS. THE MAXIMUM DEFLECTION UNDER TOTAL DEAD LOAD IS 0.403 INCHES UPWARD. THIS DEFLECTION IN TIME MIGHT BE EXPECTED TO INCREASE SLIGHTLY, BUT THIS WOULD NOT HAVE ANY DAMAGING EFFECT ON THE STRUCTURE.

ULTIMATE MOMENT

$$M_u = 1.5D + 1.8L$$
$$= 1.5 \times 2725 + 1.8 \times 1475$$
$$= 4090 + 2665$$
$$= 6755\ 'K$$
$$= 81000\ ''K$$

RESISTING ULTIMATE MOMENT

$$M_u = \phi\left[A_s f_{su}\left(d - \frac{a}{2}\right)\right] \quad \left(\text{IF } P\frac{f_{su}}{f'_c} \text{ DOES NOT EXCEED } 0.30\right)$$

THIS IS THE FORMULA FOR RECTANGULAR SECTIONS, OR FLANGED SECTIONS IN WHICH THE NEUTRAL AXIS LIES WITHIN THE FLANGE. BY INSPECTION, THE THE LATTER WILL BE OUR CASE. THIS CAN FURTHER BE CHECKED BY:

$$1.4\,dp\,\frac{f_{su}}{f'_c} < \text{FLANGE THICKNESS}$$

$A_s = 17.83$ SQ. IN.

$d = 21 + 25.5 = 46.5$ IN.

$$P = \frac{A_s}{bd} = \frac{17.83}{45 \times 46.5} = 0.0085$$

$$f_{su} = f'_s\left(1 - 0.5p\,\frac{f'_s}{f'_c}\right) \quad \text{(TENDONS ASSUMED TO BE GROUTED)}$$

$$= 160\left(1 - 0.5 \times 0.0085\ \frac{160}{5}\right)$$
$$= 138.3 \text{ K.S.I.}$$

$$1.4\,dp\,\frac{f_{su}}{f'_c} = 1.4 \times 46.5 \times 0.0083\ \frac{138.3}{5}$$
$$= 15.3 \text{ IN.} < 18 \text{ IN.}$$

$$P\,\frac{f_{su}}{f'_c} = 0.0085\ \frac{138.3}{5}$$
$$= 0.235 < 0.30$$

$$a = \frac{A_s f_{su}}{0.85 f'_c b} = \frac{17.83 \times 138.3}{0.85 \times 5 \times 45}$$
$$= 12.88 \text{ IN.}$$

$$M_u = 0.90\left[17.83 \times 138.3\ \left(46.5 - \frac{12.88}{2}\right)\right]$$

$$= 0.90 \times 17.83 \times 138.3 \times 40.06$$

$$= 88700\ ''K > 81000\ ''K$$

SHEAR

ULTIMATE SHEAR

$V_u = 1.5\,D + 1.8\,L.$

AT SUPPORT :

$V_u = 1.5 \times 155 + 1.8 \times 85$

$= 232.5 + 153.3$

$= 385.8$ K.

AT X = 10'

$V_u = 1.5 \times 130 + 1.8 \times 70$

$= 195 + 126$

$= 321$ K.

AT X = 20'

$V_u = 1.5 \times 105 + 1.8 \times 55$

$= 157.5 + 99$

$= 256.5$ K

THE SHEAR AT DIAGONAL CRACKING SHALL BE THE LESSER OF

$$V_{ci} = 0.6\, b'd \sqrt{f'_c} + \frac{M_{cr}}{\frac{M}{V} - \frac{d}{2}} + V_d$$

BUT NOT LESS THAN $1.7\, b'd\sqrt{f'_c}$

$$V_{cw} = b'd\,(3.5\sqrt{f'_c} + 0.3\, f_{pc}) + V_p$$

WHERE $b' = 18''$

$d = 46.5''$ OR $0.80 \times 54 = 43.2''$

(USE 46.5 IN ALL CASES)

$$M_{cr} = \frac{I}{y}\left(6\sqrt{f'_c} + f_{pe} - f_d\right)$$

$$\frac{I}{y} = S_B = 6.20 \times 12^3 = 10700 \text{ IN.}^3$$

$$6\sqrt{f'_c} = 6\sqrt{5000} = 425$$

	X = 0	X = 10	X = 20	
f_{pe}	1062	3827	4752	P.S.I.
f_d	0	-1597	-2916	P.S.I.
M_{cr}	15923	28400	24400	" K
$\frac{M}{V}$	0	11	25.3	FT.
V_d	155	130	105	KIPS
V_p	329	165	0	KIPS

AT X = 0

$$V_{cw} = 18 \times 46.5\left(3.5\sqrt{5000} + 0.3 \times 1062\right) + 329$$
$$= 18 \times 46.5\left(0.248^{*} + 0.319^{*}\right) + 329$$
$$= 475 + 329$$
$$= 804 \text{ K}$$

* THESE FIGURES HAVE BEEN REDUCED TO K.S.I.

AT $X = 10$

$$V_{ci} = 35.6 + \frac{28400}{132 - 23.25} + 130$$

$$= 35.6 + 261 + 130$$

$$= 426.6 \text{ K}$$

$$V_{cw} = 475 + 165$$

$$= 640 \text{ K}$$

AT $X = 20$

$$V_{ci} = 35.6 + \frac{24400}{304 - 23.25} + 105$$

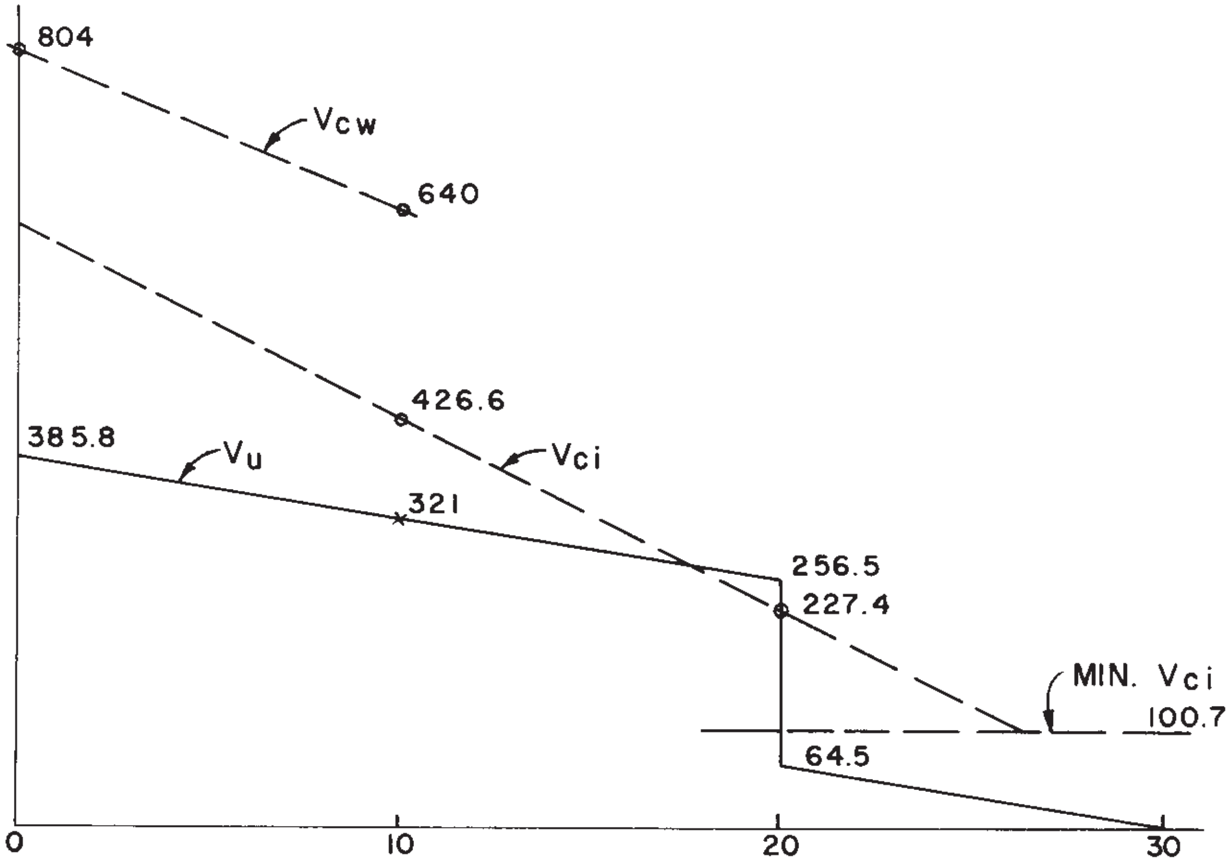
$$= 35.6 + 86.8 + 105$$

$$= 227.4 \text{ K}$$

MINIMUM V_{ci}

$$V_{ci} = 1.7 \times 18 \times 46.5 \sqrt{5000}$$

$$= 100.7$$



STEEL REQUIRED

$$A_v = \frac{(V_u - \phi\ V_c)\ S}{\phi\ d\ f_y}$$

$$= \frac{(256.5 - 0.85 \times 227.4)\ S}{0.85 \times 46.5 \times 40} = 0.04\ S.$$

BUT NOT LESS THAN

$$A_v = \frac{A_s}{80} \cdot \frac{f'_s}{f_y} \cdot \frac{S}{d} \sqrt{\frac{d}{b'}}$$

$$= \frac{17.83}{80}\ \frac{160}{40}\ \frac{S}{46.5} \sqrt{\frac{46.5}{18}} = 0.0308\ S$$

$S \leq \frac{3}{4}\ d$ OR $\leq 24''$

ASSUME # 4 ⊔ STIRRUPS

$$S = \frac{2 \times 0.196}{0.0308} = 12.75'' \quad \text{OR} \quad S = \frac{2 \times 0.196}{0.04} = 9.8''$$

USE # 4 ⊔ STIRRUPS @ 12" FULL LENGTH WITH

4 @ 9" UNDER EACH COLUMN LOAD.

END BLOCK

USUALLY AN END BLOCK IS PROVIDED IN POST-TENSIONED BEAMS, TO TRANSFER THE FORCES FROM THE BEARING PLATES TO THE FULL CONCRETE CROSS-SECTION.

THE DESIGN OF END BLOCKS IS MOSTLY EMPIRICAL. THE READER IS REFERRED TO GUYON'S "PRESTRESSED CONCRETE" FOR AN EXTENSIVE ANALYSIS OF STRESSES IN END BLOCKS OF POST-TENSIONED BEAMS.

HERE WE SHALL SET DOWN A FEW PRACTICAL RULES TO ARRIVE AT PRACTICAL DIMENSIONS AND REINFORCEMENT REQUIREMENTS.

LENGTH OF END BLOCK

USUALLY A MINIMUM OF $3/4$ OF BEAM DEPTH

$$3/4 \times 54 = 40.5 \;,\; \text{SAY } 42'' = 3'\text{-}6''$$

WIDTH OF END BLOCK

USUALLY THE WIDTH OF THE SMALLER FLANGE, AT LEAST.

USE 3'-9"

REINFORCEMENT

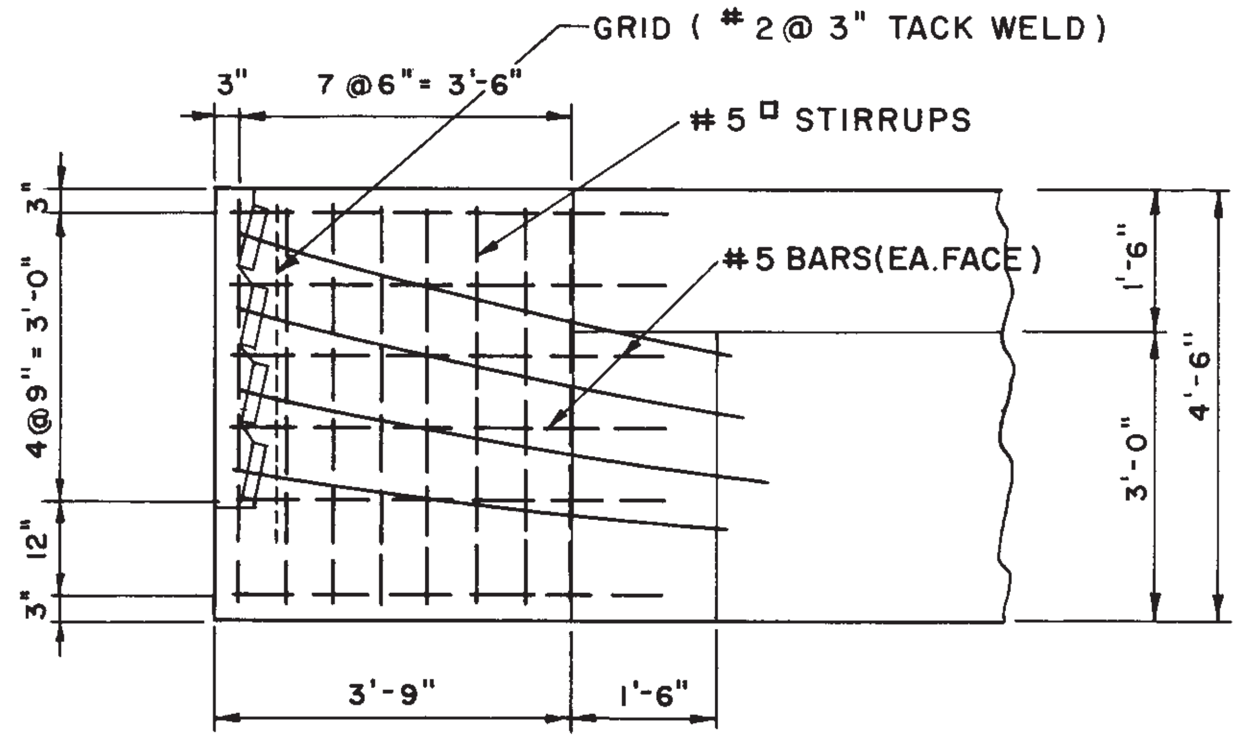
WITHOUT A COMPLETE ANALYSIS OF THE BURSTING AND SPALLING STRESSES THAT OCCUR IN AN END BLOCK, IT IS RECOMMENDED TO PROVIDE ENOUGH MILD STEEL REINFORCEMENT (STIRRUPS) TO TAKE 3% OF THE TOTAL PRESTRESSING FORCE UNDER NORMAL WORKING STRESSES (SAY 20 K.S.I.) IN OUR CASE.

$$0.03 \times 17.83 \times 112 = 60 \text{ KIPS}$$

$$A_s = \frac{60}{20} = 3.0 \text{ SQ. IN.} \quad (5\ \#5\ \square\ \text{STIRRUPS})$$

DIRECTLY UNDER THE BEARING PLATES IT IS ADVISABLE TO PROVIDE A REINFORCING GRID OF SAY

\# 2 BARS @ 3" O.C.



GRID (# 2 @ 3" TACK WELD)
3"
7 @ 6" = 3'-6"
5 □ STIRRUPS
5 BARS(EA. FACE)
3"
4 @ 9" = 3'-0"
12"
3"
1'-6"
3'-0"
4'-6"
3'-9"
1'-6"